



Low-Complexity based TDE Method for OFDM Signal in Higher Time-Varying Fading Channels

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Abstract: Orthogonal Frequency Division Multiplexing (OFDM) signal would be damaged significantly by inter-carrier interference (ICI) in higher time-varying fading channels. The ICI leads to fatal degradation of bit error rate (BER) performance due to the loss of orthogonality among subcarriers. To solve this problem, this paper proposes a high accuracy time-domain channel impulse response (CIR) estimation method and low-complexity based time-domain equalization (TDE) method for solving the simultaneous equations instead of using an inverse matrix calculation which can achieve better BER performance and lower computation complexity even in higher time-varying fading channels. The salient features of proposed method are to employ a time-domain training sequence (TS) in the estimation of channel impulse response (CIR) instead of using pilot subcarriers in the frequency domain and to employ the time domain equalization (TDE) method with maximum likelihood (ML) estimation instead of using a conventional frequency domain equalization (FDE) method. This paper also proposes a low-complexity iterative method for solving the simultaneous equations instead of using an inverse matrix calculation, which remains the computation complexity up to 7.8% of inverse matrix calculation with the same BER performance but achieves the BER performance when compared with the conventional method. This paper presents various simulation results in higher time-varying fading channels (vehicle speed \approx 381 km/hrs) to demonstrate the effectiveness of the proposed method as compared with conventional FDE and TDE methods.

Keywords: OFDM, ICI, CIR, FDE, TDE, Maximum likelihood (ML) Estimation, Training sequence (TS).

1. Introduction

Orthogonal frequency division multiplexing (OFDM) technique has been received very much attention especially in the field of wireless communications because of its efficient usage of frequency bandwidth and robustness to multipath fading channels. From these advantages, OFDM already adopted as standard transmission techniques in various wireless communications systems such as digital video broadcasting (DVB), wireless local area network (WLAN), and fourth-generation (4G) cellular systems [1-5].

The frequency-domain equalization (FDE) method is usually employed to compensate for the fading distortion in the OFDM systems under the quasi-static multipath fading channels, where

channel impulse response (CIR) in the time domain can be considered as a constant during one OFDM symbol period. In other words, the channel frequency response (CFR) which be used in the conventional FDE method and assumes constant during one OFDM symbol period. However, the CIR would be no more constant even during one OFDM symbol period in higher time-varying fading channels and causes the loss of orthogonality among subcarriers due to inter-carrier interference (ICI) [6-8]. From this fact, it is hard to compensate for the ICI by using the conventional FDE method, and the bit error rate (BER) performance of the OFDM signal would be degraded relatively in higher time-varying fading channels.

To solve the above problems, many authors have contributed many equalization methods to combat the ICI and also many methods are employed the

estimated CIR at every sampling time to compensate for the ICI [9-13]. In [9] and [10], the equalization method is operated in the frequency domain. These would lead to higher computational complexity due to the reconstruction of the inverse matrix. In [11] and [12], the ICI mitigation methods were proposed in the time domain, however, both methods using pilot subcarriers in the estimation of CIR. The CIR estimation accuracy would be degraded in higher time-varying fading channels. In [13], we proposed the ICI mitigation method both for channel estimation and TDE methods in higher time-varying fading channels. The proposed method can achieve better BER performance. However, in demodulation processing, this method requires calculating the direct inverse at every OFDM symbol which leads to high computation complexity and not suitable in a practical system.

Although the proposed equalization methods [9]-[13] can achieve better BER performance than the conventional FDE method, however, all methods require the higher CIR estimation accuracy at every sampling time. This would cause higher computation complexity for calculation inverse matrix at every OFDM symbol.

The CIR estimation usually employs scattered pilot subcarriers which inserted into data subcarriers periodically in the frequency axis. The pilot subcarriers based CIR estimation can achieve higher CIR estimation accuracy only when the number of N -point FFT/IFFT is equal to the number of M -point data subcarriers, which corresponds to the Nyquist sampling [14]. However, in the practical OFDM transmitter, a certain number of null subcarriers (zero paddings) are added at both ends of OFDM data subcarriers to reject the aliasing that occurs at the output of digital to analog (D/A) converter. Since actual CIR is occurred based on overall bandwidth including zero paddings, the estimated CIR by using the pilot subcarriers inserted within OFDM allocated bandwidth is different from actual CIR, especially when increasing the number of zero paddings. To solve this problem, we proposed a time-domain CIR estimation method by using a time-domain training sequence (TS), which can achieve higher CIR estimation accuracy even when the sampling rate of the OFDM signal is non-Nyquist sampling [13].

To reduce computation complexity and to solve the simultaneous equations for the equalization method in [9] and [13], an iterative method is employed instead of direct inverse matrix calculation. To employ an iterative method, the coefficients matrix of the simultaneous equations must be the square matrix. And to satisfy the above requirement, [9] employed the OFDM signal with guard interval

(GI) in which the coefficients matrix for the CIR over one data symbol period becomes the square matrix. On the other hand, the method in [13] employed the OFDM signal without GI in which the square coefficients matrix is generated by using the overlap and add (OLA) matrix operation after removing the interference from time-domain TS. The coefficients matrixes obtained from both [9] and [13] are entirely the same, which means that the time domain TS in [13] plays the role of GI in [9]. In [15], we proposed the TDE method with a partial differential solution for the OFDM signal without GI in quasi-static environments. The proposed TDE method shows much better BER performance than the conventional FDE method in which the data information is demodulated based on maximum likelihood (ML) estimation. From the results in [15], it can be expected that BER performance of proposed TDE without GI would have the possibility to improve BER performance further as compared with TDE methods in [9] and [13] even in higher time-varying fading channels.

To achieve superior BER performance both for quasi-static and higher time-varying fading channels, this paper firstly proposes the CIR estimation method by using the TS signal. The feature of the proposed CIR estimation method is to employ the TS signal added to each data symbol over one frame instead of conventional pilot subcarriers. Secondly, this paper proposes a low complexity equalization method for OFDM with the TS signal, which uses the CIR matrix in the time domain instead of using the CFR matrix in the conventional FDE method. The significant feature of the proposed TDE method is to employ the partial differential solution in solving the maximum likelihood (ML) equation of the expected time-domain information. As for the result of the minimization problem for solving ML equation, the coefficient matrix of the simultaneous equations in the proposed method becomes a square matrix, which can be solved iteratively without using inverse matrix calculation.

The rest of this paper is shown by the following. Section 2 introduces the problem of conventional FDE in higher time-varying fading channels. Section 3 describes the problem of Pilot subcarrier base CIR estimation in non-Nyquist sampling and also proposes a CIR estimation method by using the TS signal. Section 4 proposes the TDE method and low-complexity based TDE method in conjunction with estimated CIR at every sampling time. Section 5 presents different computer simulation results to verify the effectiveness of the proposed method as compared with conventional FDE methods. Section 6 draws some conclusions.

2. Problem of conventional FDE method in the higher time-varying fading channel

2.1 OFDM signal and channel models

In OFDM systems, input data are encoded by a forward error correction (FEC) code [16]. The M encoded data are modulated by quadrature amplitude modulation (QAM) method. The OFDM signal $X_D(m,k)$ at k -th subcarriers of an m -th symbol is mapped into a certain frequency band with M subcarriers continuously from subcarrier number N_{Z1} to N_{Z2} ($N_{Z2} - N_{Z1} + 1 = M$) within N subcarriers. The null subcarriers (zero paddings) are added at both ends of M data subcarriers. The data information over N subcarriers is given by,

$$X_Z(m, k_1) = \begin{cases} 0 \text{ (Zero padding)}, & 0 \leq k_1 \leq N_{Z1} - 1 \\ X_D(m, k_1 - N_{Z1}), & N_{Z1} \leq k_1 \leq N_{Z2} \\ 0 \text{ (Zero padding)}, & N_{Z2} + 1 \leq k_1 \leq N \end{cases} \quad (1)$$

where $(N-M)$ is the number of zero paddings (N_Z) added at both ends of M data subcarriers in the frequency domain. After added zero paddings, $X_Z(m, k_1)$ is converted into the time domain signal by mean of N -point inverse fast Fourier transform (IFFT) which can be given by,

$$x(m, n_1) = \frac{1}{\sqrt{N}} \cdot \sum_{k_1=0}^{N-1} X_Z(m, k_1) \cdot e^{j \frac{2\pi n_1 k_1}{N}} \quad (2)$$

$, 0 \leq n_1 \leq N - 1$

where $x(m, n_1)$ is the transmitted OFDM signal in the time domain at an n_1 -th sampling time of m -th symbol. For compensation of inter symbol interference (ISI), introduced by signal propagation through multipath fading channels, the OFDM signal employs the guard interval (GI) which is a copy of the last N_g sampling time of data symbol inserted into the front of the signal [17] which can be expressed by,

$$x_G(m, n_2) = \begin{cases} x(m, n_2 + N - N_g), & 0 \leq n_2 \leq N_g - 1 \\ x(m, n_2 - N_g), & N_g \leq n_2 \leq N + N_g - 1 \end{cases} \quad (3)$$

In higher time-varying fading channels such as high-speed vehicles or trains communications [18], [19] the channel communication is often modeled as the Rician multipath fading channels. Here, the channel impulse response (CIR) $h_l(m, n_2)$ for l -th

delay path at an n_2 -th sampling time of m -th symbol in time-varying fading channels can be given by,

$$h_l(m, n_2) = \sum_{l=0}^{L-1} h_l(m) \cdot \delta(n_2 - \tau_l) \quad (4)$$

where $h_l(m)$ is the complex amplitude of CIR for l -th delay path of an m -th symbol.

2.2 Problem of conventional FDE method

Assuming that the length of GI is longer than the maximum delay paths in the wireless channel and perfect synchronization timing is considered at the receiver. The received time-domain signal after removing GI at the n_2 -th sample time during the m -th symbol can be expressed by,

$$\begin{aligned} r(m, n_2) &= \sum_{l=0}^{L-1} h_l(m, n_2) \otimes x_G(m, n_2) + w(m, n_2), \\ &= \sum_{l=0}^{L-1} h_l(m) \cdot x_G(m, n_2 - N_g - l) + w(m, n_2), \end{aligned} \quad (5)$$

$N_g \leq n_2 \leq N + N_g - 1$

where \otimes represents the convolution. $r(m, n_2)$ is the received time-domain signal without GI and $w(m, n_2)$ is the zero-mean additive white Gaussian noise (AWGN) with variance σ^2 . The $x_G(m, n_2 - N_g - l)$ in Eq. (5) corresponds to $x(m, n_1 - l)$, which given in Eq. (2) satisfies the followings,

$$\begin{cases} n_1 - l \geq 0, & x(m, n_1 - l) = x(m, n_1 - l) \\ n_1 - l < 0, & x(m, n_1 - l) = x(m, N - l) \end{cases} \quad (6)$$

The received time-domain signal $r(m, n_2)$ in Eq. (5) is converted to the frequency domain by N -point FFT which can be represented by,

$$\begin{aligned} R(m, k_1) &= \sum_{n_2=0}^{N-1} r(m, n_2) \cdot e^{-j \frac{2\pi k_1 n_2}{N}} \\ &= X_Z(m, k_1) \cdot H_m(k_1, k_1) \\ &+ \underbrace{\sum_{\substack{k_1=0 \\ k_1 \neq k_1}}^{N-1} X_Z(m, k_1) \cdot H_m(k_1, k_1)}_{ICI} \\ &+ W(m, k_1), \quad 0 \leq k_1 \leq N - 1 \end{aligned} \quad (7)$$

where $X_Z(m, k_1)$ is the transmitted data information, which given in Eq. (1) and H_m is the channel frequency response (CFR). From Eq. (7), it can be seen that the second term is the ICI causes from time-varying fading channels. Fig. 1 shows a schematic

diagram for the relationships between the CIR in the time domain and the channel frequency response (CFR) in the frequency domain. In the figure, the CFR is obtained by converting the CIR at a certain sampling time during one OFDM symbol period by using a discrete Fourier transform (DFT). Fig. 1.1 and 1.2 show the CIR and CFR in both quasi-static channels and high time-varying channels. From the figure, the CFR in high time-varying fading channels is changing according to the CIR at the different sampling times. From this fact, it is difficult to compensate for the time-varying fading distortion by using the conventional FDE method.

3. CIR Estimation methods

3.1 Problem of conventional CIR estimation method

In the conventional CIR estimation method, the pilot subcarrier usually employed, which be inserted periodically in frequency and time axis. The received frequency domain signal of an m -th symbol at k_1 -th subcarrier in Eq. (7) can be rewritten by,

$$R(m, k_1) = X_Z(m, k_1) \cdot H(m, k_1) + W(m, k_1) \quad (8)$$

Here, the estimated CFR of the m -th symbol at k_p -th pilot subcarrier can be expressed by,

$$\hat{H}(m, k_p) = \frac{R(m, k_p)}{P(m, k_p)} \quad (9)$$

where $\hat{H}(m, k_p)$ is the estimated CFR, $R(m, k_p)$ received frequency-domain pilot subcarriers and $P(m, k_p)$ is the pilot pattern as known at the receiver. By using Eq. (9) with assumed that the $\hat{h}_l(m)$ is the unknown parameter, the estimated CFR is given by,

$$\hat{H}(m, k_p) = \sum_{l=0}^{L-1} \hat{h}_l(m) \cdot e^{-j \frac{2\pi l k_p}{N}} \quad (10)$$

By using Eqs. (9) and (10), the estimated CIR $\hat{h}_l(m)$ at every pilot symbol can be estimated by solving ML equation [17] under the constraint condition error between $\hat{H}(m, k_p)$ in Eqs. (9) and (10) is minimized as

$$Y[h_0, h_1, \dots, h_{N_g-1}] = \arg \min \left[\sum_{p=0}^{N_p-1} \left| \hat{H}(m, k_p) - \sum_{l=0}^{N_g-1} \hat{h}_l(m) \cdot e^{-j \frac{2\pi l k_p}{N}} \right|^2 \right] \quad (11)$$

where N_p is the number of pilot subcarriers inserted

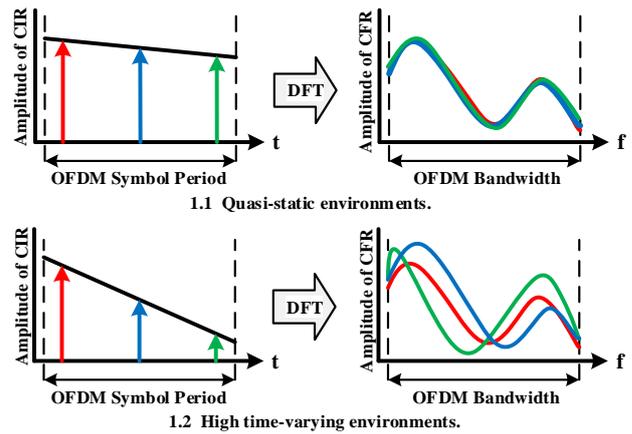


Figure. 1 Relationships between CIR and CFR

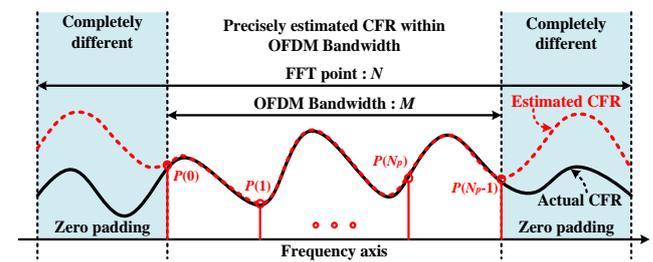


Figure. 2 Comparison of estimated CFR and actual CFR

in one OFDM symbol. Since the dependence from optimization parameters $\hat{h}_l(m)$ is linear in Eq. (11), the solution can be realized by the following Moore-Penrose generalized matrix inversion.

$$\underbrace{[\hat{h}_l(m)]}_{N_g \times 1} = \dagger \underbrace{[Z(k_p, l)]}_{N_g \times N_p} \cdot \underbrace{[\hat{H}(m, k_p)]}_{N_p \times 1} \quad (12)$$

where \dagger is the Moore-Penrose inverse matrix and $[Z(k_p, l)]$ is given by,

$$[Z(k_p, l)] = e^{-j \frac{2\pi l k_p}{N}} \quad (13)$$

From Eqs. (10) to (13), there is a severe problem in the accuracy of CIR estimation from the received OFDM signal. Fig. 2 shows a comparison of the estimated CFR and actual CFR. The CFR can be obtained by performing a discrete Fourier transform (DFT) to CIR at a certain sampling time during one OFDM symbol period. In the Fig. 2, it can be observed that the estimated CFR by using pilot subcarriers can be estimated CFR precisely within OFDM bandwidth. However, the actual CFR in the real channel is entirely different from the estimated CFR. From this fact, the actual CIR is also completely different from the estimated CIR when the number of N -points FFT/IFFT is not equal to the

number of M data subcarriers, which corresponds to non-Nyquist sampling [13].

3.2 Proposed CIR estimation method

Fig. 3 shows the structure of the transmitter for the proposed method. At the transmitter side, the input data sequence is modulated. Then, added zero paddings and converted to the time domain signal $x(m, n_1)$ as same as the conventional OFDM signal. Fig. 4 shows the frame format employed in the proposed method. The time-domain TS1 and TS2 with the length of N_{TS} are added at both ends of every data symbol and would be used in the estimation of CIR at every data symbol. The TS signal also used as the role of GI to avoid the ISI occurred in multipath fading channels. The transmitted time-domain signal $x_T(m, n_2)$ including both the TS1 and TS2 can be expressed by,

$$x_T(m, n_2) = \begin{cases} d_1(m, n_2), & 0 \leq n_2 \leq N_{TS} - 1 \\ x(m, n_2 - N_{TS}), & N_{TS} \leq n_2 \leq N + N_{TS} - 1 \\ d_2(m, n_2 - N - N_{TS}), & N + N_{TS} \leq n_2 \leq N + 2N_{TS} - 1 \end{cases} \quad (14)$$

where $d_1(m, n)$ and $d_2(m, n)$ are the time domain TS1 and TS2 with the length of N_{TS} samples ($0 \leq n \leq N_{TS} - 1$) of which data patterns are known at the receiver. For simplicity, this paper assumes the data patterns both for TS1 and TS2 are assumed to be the same as $d(m, n)$. The length of N_{TS} should be taken longer than the length of delay paths (L) as the same as the role of GI to avoid ISI. Here it is assumed that $h_l(m)$ is the constant during the period of TS1 even in higher time-varying fading channels. The actual received TS signal passed through the multi-path fading channels can be expressed by,

$$r_{TS}(m, n_2) = \sum_{l=0}^{L-1} h_l(m) \cdot x_T(m, n_2 - l) + w(m, n_2), \quad (15) \\ 0 \leq n_2 \leq N_{TS} - 1$$

where $h_l(m)$ is the ideal time-domain CIR for the l -th delay path at the m -th symbol. Assuming that the estimated CIR $\hat{h}_l(m)$ is unknown parameters, the expected received time-domain TS can be given by,

$$\hat{r}_{TS}(m, n_2) = \sum_{l=0}^{N_{TS}-1} \hat{h}_l(m) \cdot d(m, n_2 - l), \quad (16) \\ 0 \leq n_2 \leq N_{TS} - 1$$

where $\hat{h}_l(m)$ is the estimated time-domain CIR for the

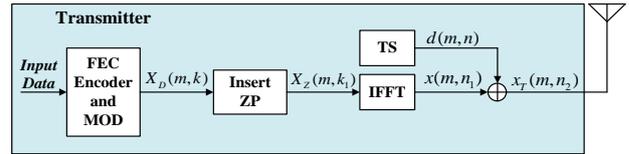


Figure. 3 Structure of the proposed TDE method

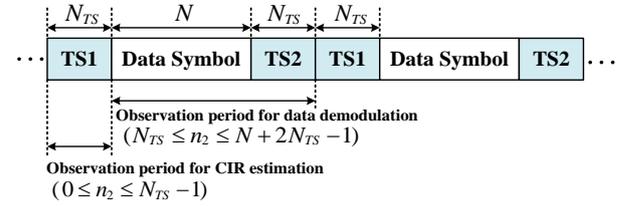


Figure. 4 Proposed frame format in the time domain

l -th delay path at the m -th symbol. The estimated CIR $\hat{h}_l(m)$ can be estimated by solving the following ML equation under constraint with minimizing the difference of actual received TS signal $r_{TS}(m, n_2)$ in Eq. (15) and expected received signal $\hat{r}_{TS}(m, n)$ in Eq. (16).

$$\Upsilon = \arg \min_{\hat{h}_l(m)} \left[\sum_{n_2=0}^{N_{TS}-1} |r_{TS}(m, n_2) - \hat{r}_{TS}(m, n_2)|^2 \right] \quad (17)$$

The following equation can simply solve the ML equation in Eq. (17),

$$r_{TS}(m, n_2) - \hat{r}_{TS}(m, n_2) = 0, \quad 0 \leq n_2 \leq N_{TS} - 1 \quad (18)$$

In the result of Eq. (18), by minimizing the difference of actual received TS signal and expected received TS signal can be expressed by,

$$r_{TS}(m, n_2) = \sum_{l=0}^{N_{TS}-1} \hat{h}_l(m) \cdot d(m, n_2 - l), \quad 0 \leq n_2 \leq N_{TS} - 1 \quad (19)$$

Finally, Eq. (19) can be solved as the results on the following simultaneous equations.

$$\underbrace{[\hat{h}_l(m)]}_{N_{TS} \times 1} = \underbrace{[d(m, n_2 - l)]}_{N_{TS} \times N_{TS}}^{-1} \cdot \underbrace{[r_{TS}(m, n_2)]}_{N_{TS} \times 1} \quad (20)$$

where $[\cdot]^{-1}$ is matrix inversion operator and $d(m, n_2 - l)$ given in Eq. (20) satisfies the followings,

$$\begin{cases} n_2 - l \geq 0, & d(m, n_2 - l) = d(m, n_2 - l) \\ n_2 - l < 0, & d(m, n_2 - l) = d(m, N_{TS} - l) \end{cases} \quad (21)$$

where $d(m, n)$ is the time domain TS and its data pattern is known at the receiver. The inverse matrix $[d(m, n_2 - l)]^{-1}$ in Eq. (20) can be calculated in advance and leads to the considerable reduction of computation complexity in the estimation of CIR at every symbol. By using the estimated $[\hat{h}_l(m)]$ at every

symbol, the estimated CIR at every sampling time $[\hat{h}_i(m, n_2)]$ can be estimated by using the cubic spline interpolation method [20] between two consecutive estimated CIRs of $[\hat{h}_i(m)]$ and $[\hat{h}_i(m+1)]$.

Fig. 5 shows a schematic diagram for the cubic spline interpolation method to estimate the CIR at every sampling time. The estimation accuracy of CIR at every sampling time for the proposed method will be evaluated in section 5 as compared with the conventional pilot base CIR estimation when the sampling rate of the OFDM signal is a non-Nyquist sampling.

4. Time domain equalization methods

4.1 TDE with OLA method

The proposed equalization methods in [9] and [13] show almost the same BER performance when using the ideal CIR $h_i(m, n_2)$ at every sampling time for data demodulation. However, [9] considers the estimated CIR $\hat{h}_i(m, n_2)$ at every sampling time of using the pilot subcarriers based CIR estimation method. The estimation accuracy of CIR at every sampling time is no more satisfied as described in section 3. On the other hand, in [13] using the time domain TS for estimation of CIR at every sampling time, which can be estimated CIR very precisely even when the sampling rate of OFDM signal is the non-Nyquist sampling and also even in higher time-varying fading channels [13].

Fig. 6 shows the structure of the receiver for the TDE with the OLA method. By assuming the ideal CIR $h_i(m, n_2)$ at every sampling time, the received data signal $r_D(m, n_2)$ at every sampling time during the observation period for data demodulation from N_{TS} to $N+2N_{TS}-1$ can be expressed by,

$$r_D(m, n_2) = \sum_{l=0}^{L-1} h_i(m, n_2) \cdot x_T(m, n_2 - l) + w(m, n_2), \quad (22)$$

$$N_{TS} \leq n_2 \leq N + 2N_{TS} - 1$$

where $x_T(m, n_2 - l)$ given in (14) satisfies the followings,

$$\begin{cases} n_2 - l \leq N_{TS} - 1, & x_T(m, n_2 - l) = d(m, n_2 - l) \\ n_2 - l \geq N + N_{TS}, & x_T(m, n_2 - l) = d(m, n_2 - N - N_{TS} - l) \end{cases} \quad (23)$$

The actual received data signal in Eq. (22) includes the interferences of TS, which are added at the start and the end of data symbol from the TS1 and TS2, respectively. By using the estimated CIR $\hat{h}_i(m, n_2)$ at every sampling time and the data pattern of $d(m, n_2)$ both for TS1 and TS2 which is known at the receiver,

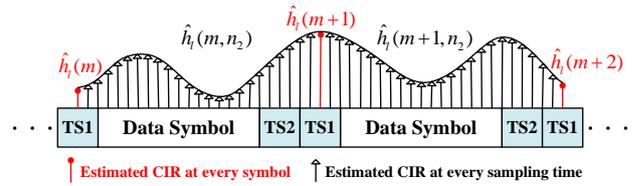


Figure. 5 Estimation of CIR at every sampling time by using a cubic spline interpolation method

the interference of TS added at both ends of the time-domain data signal can be removed by the following equation.

$$r_F(m, n_2) = \begin{cases} r_D(m, n_2) - \sum_{l=n_2-N_{TS}+1}^{N_{TS}-1} \hat{h}_i(m, n_2) \cdot d(m, n_2 - l), & (N_{TS} \leq n_2 \leq 2N_{TS} - 2) \\ r_D(m, n_2), & (2N_{TS} - 1 \leq n_2 \leq N + N_{TS} - 1) \\ r_D(m, n_2) - \sum_{l=0}^{n_2-N_{TS}} \hat{h}_i(m, n_2) \cdot d(m, n_2 - N - N_{TS} - l), & (N + N_{TS} \leq n_2 \leq N + 2N_{TS} - 2) \end{cases} \quad (24)$$

where $r_F(m, n_2)$ is the received time-domain signal after removing the interference of TS from the actually received signal $r_D(m, n_2)$ in Eq. (22). When the transmitted time-domain data $x(m, n_1)$ given in Eq. (2) is assumed as the unknown parameters, the expected time domain received data $\hat{r}_E(m, n_2)$ without the interference of TS which corresponds to Eq. (24) can be given by,

$$\hat{r}_E(m, n_2) = \begin{cases} \sum_{l=0}^{n_2-N_{TS}} \hat{h}_i(m, n_2) \cdot \hat{x}(m, n_2 - N_{TS} - l), & (N_{TS} \leq n_2 \leq 2N_{TS} - 2) \\ \sum_{l=0}^{N_{TS}-1} \hat{h}_i(m, n_2) \cdot \hat{x}(m, n_2 - N_{TS} - l), & (2N_{TS} - 1 \leq n_2 \leq N + N_{TS} - 1) \\ \sum_{l=n_2-N-N_{TS}+1}^{N_{TS}-1} \hat{h}_i(m, n_2) \cdot \hat{x}(m, n_2 - N_{TS} - l), & (N + N_{TS} \leq n_2 \leq N + 2N_{TS} - 2) \end{cases} \quad (25)$$

The maximum likelihood (ML) can estimate the unknown parameter of time-domain data $\hat{x}(m, n_1)$, that ML equation solve under the constraint with minimizing the difference between the actually received data $r_F(m, n_2)$ in Eq. (24) and the expected received data $\hat{r}_E(m, n_2)$ in Eq. (25).

$$Y = \arg \min_{\hat{x}(m, n_1)} \left[\sum_{n_2=N_{TS}}^{N+2N_{TS}-2} |r_F(m, n_2) - \hat{r}_E(m, n_2)|^2 \right] \quad (26)$$

The following equations can solve the ML equation in Eq. (26).

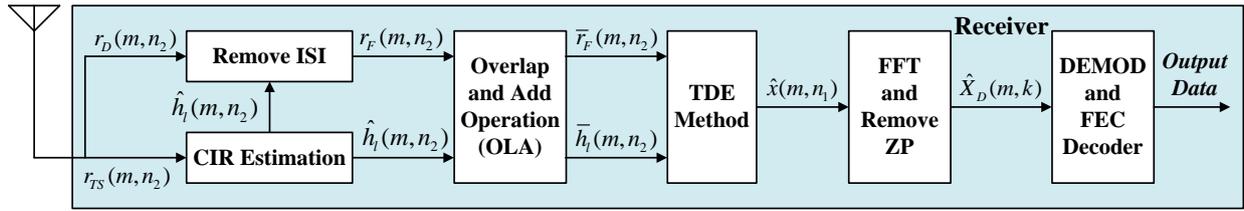


Figure. 6 Receiver structure for TDE with the OLA method

$$r_F(m, n_2) - \hat{r}_E(m, n_2) = 0, \quad N_{TS} \leq n_2 \leq N + N_{TS} - 1 \quad (27)$$

By using the condition in Eq. (27) and substituting Eq. (25) into Eq. (27), Eq. (27) can be expressed by the following simultaneous equations.

$$r_F(m, n_2) = \begin{cases} \sum_{l=0}^{n_2 - N_{TS}} \hat{h}_l(m, n_2) \cdot \hat{x}(m, n_2 - N_{TS} - l), & N_{TS} \leq n_2 \leq 2N_{TS} - 2 \\ \sum_{l=0}^{N_{TS}-1} \hat{h}_l(m, n_2) \cdot \hat{x}(m, n_2 - N_{TS} - l), & 2N_{TS} - 1 \leq n_2 \leq N + N_{TS} - 1 \\ \sum_{l=n_2 - N - N_{TS} + 1}^{N_{TS}-1} \hat{h}_l(m, n_2) \cdot \hat{x}(m, n_2 - N_{TS} - l), & N + N_{TS} \leq n_2 \leq N + 2N_{TS} - 2 \end{cases} \quad (28)$$

where $\hat{x}(m, n_2 - N_{TS} - l)$ in Eq. (28) correspond to $\hat{x}(m, n_1 - l)$ with condition $(0 \leq n_1 \leq N - 1)$. Finally, Eq. (28) is resulted in the following matrix operation.

$$\underbrace{[r_F(m, n_2)]}_{(N+N_{TS}-1) \times 1} = \underbrace{[\hat{h}_l(m, n_2)]}_{(N+N_{TS}-1) \times N} \cdot \underbrace{[\hat{x}(m, n_1)]}_{N \times 1} \quad (29)$$

where $[r_F(m, n_2)]$ received a time-domain signal with a length of $(N+N_{TS}-1) \times 1$ matrix. $[\hat{h}_l(m, n_2)]$ is time-domain CIR with a length of $(N+N_{TS}-1) \times N$ matrix. Here $[r_F(m, n_2)]$ in Eq. (29) can be spilled into its upper $N \times 1$ part and it is lower $(N_{TS}-1) \times 1$ part. Similarly, $[\hat{h}_l(m, n_2)]$ can be split into its upper $N \times N$ part and its lower $(N_{TS}-1) \times N$ part. By adding the lower elements to upper elements, $[r_F(m, n_2)]$ and $[\hat{h}_l(m, n_2)]$ can be given by the following matrixes with the size of $N \times 1$ and $N \times N$, respectively.

$$[\bar{r}_F(m, n_2)] = \Phi[r_F(m, n_2)] = \begin{bmatrix} I_N & I_{N_{TS}-1} \\ & 0 \end{bmatrix} \cdot [r_F(m, n_2)] \quad (30)$$

$$[\bar{h}_l(m, n_2)] = \Phi[\hat{h}_l(m, n_2)] = \begin{bmatrix} I_N & I_{N_{TS}-1} \\ & 0 \end{bmatrix} \cdot [\hat{h}_l(m, n_2)] \quad (31)$$

where Φ denotes an overlap-and-add (OLA) operator which makes circular convolution feasible. From Eqs. (30) and (31), Eq. (29) becomes the following matrix operations with the size of the $N \times N$ matrix.

$$\underbrace{[\bar{r}_F(m, n_2)]}_{N \times 1} = \underbrace{[\bar{h}_l(m, n_2)]}_{N \times N} \cdot \underbrace{[\hat{x}(m, n_1)]}_{N \times 1} \quad (32)$$

From Eq. (32), it can be observed that the CIR matrix becomes the circular matrix, which is the same as the OFDM signal with GI when assuming the quasi-static multipath fading channels [13]. The expected time-domain signal can be estimated by using the direct calculation of the inverse matrix which is given by,

$$\underbrace{[\hat{x}(m, n_1)]}_{N \times 1} = \underbrace{[\bar{h}_l(m, n_2)]^{-1}}_{N \times N} \cdot \underbrace{[\bar{r}_F(m, n_2)]}_{N \times 1} \quad (33)$$

where $[\bar{h}_l(m, n_2)]^{-1}$ is the inverse of the channel matrix and $[\hat{x}(m, n_1)]$ is the estimated time-domain data information matrix. Since the TDE with the OLA method can track the time-varying CIR at every sampling time as given in Eq. (28), it is possible to mitigate the ICI and expected to achieve better BER performance than the conventional FDE method in higher time-varying fading channels. In Eq. (33), the estimated time-domain $\hat{x}(m, n_1)$ can be solved simply by using the inverse matrix calculation with the size of $N \times N$. The computational complexity of the TDE with the OLA method can be considered as matrix inversion is $O(N^3)$, and matrix multiplication is $O(N^2)$. Here, the total computational complexity for the TDE with the OLA method is to order $N^3 + N^2$ at every data symbol.

4.2 Proposed TDE with partial differential method

In the TDE with the OLA method, the BER performance would be degraded from OLA operator in (30) and (31) due to the summation of unexpected phase noise from AWGN and multipath channel due to its upper $N \times 1$ and lower $(N_{TS}-1) \times 1$ parts. Fig. 7 shows the structure of the receiver for the proposed TDE with the partial differential method. The

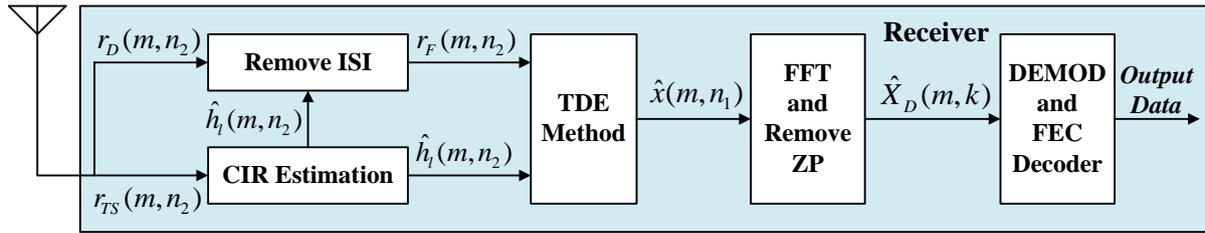


Figure. 7 Receiver structure for proposed TDE with the partial differential method

maximum likelihood (ML) can estimate the unknown parameter of time-domain data $\hat{x}(m, n_1)$ in Eq. (2), that ML equation solve under the constraint with minimizing the difference between the actually received data $r_F(m, n_2)$ in Eq. (24) and the expected received data $\hat{r}_E(m, n_2)$ in Eq. (25).

$$\Upsilon = \arg \min_{\hat{x}(m, s)} \left[\sum_{n_2=N_{TS}}^{N+2N_{TS}-2} |r_F(m, n_2) - \hat{r}_E(m, n_2)|^2 \right] \quad (34)$$

The ML equation (34) can be solved by taking the partial differentiation for all unknown parameters of $\hat{x}^*(m, s)$ which can be expressed by,

$$\frac{\partial \Upsilon}{\partial \hat{x}^*(m, s)} = \frac{\partial \left(\sum_{n_2=N_{TS}}^{N+2N_{TS}-2} |r_F(m, n_2) - \hat{r}_E(m, n_2)|^2 \right)}{\partial \hat{x}^*(m, s)} = 0, \quad (35)$$

$(0 \leq s \leq N-1)$

where * represents the conjugate complex number. By using Eq. (35), the ML equation (34) can be expressed by the following simultaneous equations with N unknown parameters of $\hat{x}(m, n_1)$.

$$\underbrace{[b(m, s)]}_{N \times 1} = \underbrace{[A_m(s, n_1)]}_{N \times N} \cdot \underbrace{[\hat{x}(m, n_1)]}_{N \times 1} \quad (36)$$

where $b(m, s)$ and $A_m(s, n_1)$ can be expressed by,

$$b(m, s) = \sum_{n_2=N_{TS}}^{N+2N_{TS}-2} r_F(m, n_2) \frac{\partial \hat{r}_E^*(m, n_2)}{\partial \hat{x}^*(m, s)}, \quad 0 \leq s \leq N-1 \quad (37)$$

$$A_m(s, n_1) = \sum_{n_2=N_{TS}}^{N+2N_{TS}-2} \hat{r}_E(m, n_2) \frac{\partial \hat{r}_E^*(m, n_2)}{\partial \hat{x}^*(m, s)}, \quad 0 \leq n_1 \leq N-1 \quad (38)$$

where,

$$s = n_2 - N_{TS} - l \quad \text{in} \quad \frac{\partial \hat{r}_E^*(m, n_2)}{\partial \hat{x}^*(m, s)} \quad (39)$$

$$n_1 = n_2 - N_{TS} - l \quad \text{in} \quad \hat{r}_E(m, n_2) \quad (40)$$

From Eq. (36), the unknown parameters $\hat{x}(m, n_1)$ can be solved by using the inverse matrix of $[A_m(s, n_1)]$ which is given by,

$$\underbrace{[\hat{x}(m, n_1)]}_{N \times 1} = \underbrace{[A_m(s, n_1)]^{-1}}_{N \times N} \cdot \underbrace{[b(m, s)]}_{N \times 1} \quad (41)$$

Similar to the TDE with OLA method, the estimated time-domain signal $\hat{x}(m, n_1)$ in Eq. (41), the M encoded data information $\hat{X}_D(m, k)$ can be demodulated by processing N -points FFT, remove zero paddings and demodulation which are all the opposite processing at the transmitter side. Here, the total computational complexity for proposed TDE with the partial differential method is order N^3+N^2 at every OFDM symbol. From Eqs. (33) and (41), The computational complexity for both proposed TDE with OLA and proposed TDE with partial differential methods is significantly high and is not suitable for practical implementation because it is required to calculate the inverse matrix at every data symbol.

4.3 Proposed TDE with iterative method

The coefficients matrix $[A_m(s, n_1)]$ after taking the partial differentiation in Eq. (36) which is the square matrix with a size of $N \times N$. This means that matrix $[A_m(s, n_1)]$ can be solved by using an iterative calculation. The conjugate gradient squared (CGS) algorithm [21] is well known as one of the iterative methods which can solve a system of N linear equations with N unknown parameters. Let us consider the system $A\hat{x}=b$, where A corresponds to $A_m(s, n_1)$ in Eq. (36) and it is the Hermitian and positive definite matrix with the size of $N \times N$. The exact CGS solution can be obtained after at most N steps. Hence, stopping the iteration after $N_{iter} (< N)$ steps would yield an approximate solution. In the demodulation of every data symbol, the CGS algorithm minimizes iteratively calculates the cost function in a reduced-rank Krylov subspace. When the spectral condition number of the matrix A is too high, a preconditioned matrix D is employed which is called the precondition CGS (PCGS) algorithm. The PCGS algorithm solves the system by,

$$D^{-1}A\hat{x} = D^{-1}b \quad (42)$$

Table 1. Preconditioned conjugate gradient squared (PCGS) algorithm

PCGS Algorithm	Complexity
Initialization :	
\hat{x}_0 is an initial guess;	
$r_0 = b - A\hat{x}_0$;	$O(N^2)$
$\tilde{r}_0 = r_0$; $\rho_0 = (r_0, \tilde{r}_0)$;	$O(N)$
$\beta_{-1} = \rho_0$; $p_{-1} = q_0 = 0$;	$O(N)$
1. for $i = 0, 1, 2, \dots, N_{\max}$ do	
2. $u_i = r_i + \beta_{i-1} q_i$;	$O(N)$
3. $p_i = u_i + \beta_{i-1} (q_i + \beta_{i-1} p_{i-1})$;	$O(N)$
4. $\hat{p} = D^{-1} p_i$;	$O(N)$
5. $\hat{v} = A\hat{p}_0$;	$O(N^2)$
6. $\alpha_i = \rho_i / (\tilde{r}_0, \hat{v})$;	$O(N)$
7. $q_0 = u_i - \alpha_i \hat{v}$;	$O(N)$
8. $\hat{u} = D^{-1} (u_i + q_{i+1})$;	$O(N)$
9. $\hat{x}_{i+1} = \hat{x}_i + \alpha_i \hat{u}$;	$O(N)$
10. if $NMSE$ in (44) > Threshold level (TOL)	$O(N)$
11. return; else	
12. $r_{i+1} = r_i - \alpha_i A\hat{u}$;	$O(N^2)$
13. $\rho_{i+1} = (r_0, \tilde{r}_{i+1})$;	$O(N)$
14. $\alpha_i = \rho_{i+1} / \rho_i$;	$O(N)$
15. end for	

where the inversion of matrix D should be a computationally efficient operation. For the rest of our analysis, we assume for simplicity that matrix $D(m)$ is the diagonal matrix of $A_m(s, n_1)$ at m -th symbol. The initial solution of $\hat{x}(m, n_1)$ is given by,

$$\underbrace{[\hat{x}(m, n_1)]^{(0)}}_{N \times 1} = \underbrace{[D(m)]^{-1}}_{N \times N} \cdot \underbrace{[b(m, s)]}_{N \times 1} \quad (43)$$

The reduced-rank PCGS method is described analytically in Table 1. The repetition of the PCGS algorithm is stopped when the following normalized mean square error ($NMSE$) between the i -th and $(i+1)$ -th solutions of $[\hat{x}(m, n_1)]$ is smaller than the predetermined threshold level (TOL).

$$NMSE = \frac{\sum_{n_1=0}^{N-1} \left| [\hat{x}(m, n_1)]^{(i)} - [\hat{x}(m, n_1)]^{(i+1)} \right|^2}{\sum_{n_1=0}^{N-1} \left| [\hat{x}(m, n_1)]^{(i)} \right|^2} \quad (44)$$

In the proposed iterative based TDE with the PCGS algorithm, the following procedures are repeated up to either the number of iterations reaches to the predetermined maximum number (N_{\max}) or $NMSE$ values less than the predetermined threshold level (TOL).

Step1: The maximum iteration number is set to N_{\max} ($N_{\max}=5$), the threshold level is set to TOL , and the initial solution of $[\hat{x}(m, n_1)]^{(0)}$ is given by Eq. (43).

Step2: Calculate the i -th solution of $[\hat{x}(m, n_1)]^{(i)}$ by the PCGS algorithm and calculate the $NMSE$ by Eq. (44).

Step3: Compare the $NMSE$ obtained at the i -th iteration with the predetermined threshold level of TOL . If the $NMSE$ is less than TOL , the $[\hat{x}(m, n_1)]^{(i)}$ is output as the estimated data information. If not, repeat the same procedures. If the number of iterations reaches to predetermined N_{\max} , $[\hat{x}(m, n_1)]^{(N_{\max})}$ is output as the estimated data information.

From Table 1, it is clear that the computational complexity is determined by the initialization of line 5 and line 12. Specifically, the total computational complexity order for the demodulation of each OFDM symbol can be expressed by $N_{\text{iter}}(2N^2+10N)+N^2+2N$. The following equation defines the ratio of computation complexity between TDE with the inverse matrix and TDE with an iterative method for the whole OFDM frame.

$$R = \frac{N_S \cdot \{N_{\text{Aver}} \cdot (2N^2 + 10N) + N^2 + 2N\}}{N_S \cdot \{N^3 + N^2\}} \quad (45)$$

$$= \frac{N_{\text{Aver}} \cdot (2N^2 + 10N) + N^2 + 2N}{N^3 + N^2}$$

where N_S is the number of data symbols in one OFDM frame, and N_{Aver} is the average number of iterations in one OFDM frame.

5. Performance evaluation

This section presents various simulation results to verify the effectiveness of proposed TDE with the iterative method as compared with the conventional FDE method [14] and the TDE with the OLA method [13]. Table 2 shows the list of simulation parameters, which use the following evaluations. The conventional FDE method employs pilot subcarriers with an interval of pilot subcarriers in the frequency axis is $FIP=4$, and the pilot symbol includes the data subcarriers, which added at every time axis ($TIP=1$).

The proposed CIR estimation method is employed both of the proposed TDE with iterative and TDE with OLA methods. The communication channel is modeled by the Rician multipath fading, which usually experienced by the user on the higher moving vehicle [18]. We assume the normalized Doppler frequency $R_D = f_{d\max} / \Delta f$ (%), which is defined by the ratio of maximum Doppler frequency $f_{d\max}$ to the subcarrier spacing Δf is employed as the measure of mobile condition.

Fig. 8 shows the time domain CIR estimation accuracy at every sampling time, which evaluated by the normalized mean square error (NMSE) for the

pilot subcarriers and the proposed TS methods when changing the number of zero paddings both sides of data subcarriers (N_Z). From the simulation results in Fig. 8, it shows that the proposed TS method can keep the higher estimation accuracy regardless of the number of N_Z . While the estimation accuracy for the pilot subcarriers method is getting worse when increasing the number of N_Z and R_D , respectively.

Fig. 9 shows BER performance for proposed TDE with iterative method when changing the threshold level (TOL). In the simulation, normalized Doppler frequency R_D is 15%, which corresponds to the vehicle speed of 286 km/hrs. In Fig. 9, it can be observed that the BER performance of proposed TDE with the iterative method is very close to that for the inverse matrix calculation method when the threshold level TOL is less than 0.01 with regardless of operation C/N. From the results in Fig. 9, the threshold level TOL for the NMSE is taken by $TOL=0.01$ in the next following evaluations.

Table 2. Simulation parameters

Parameter		Value
Number of FFT points (N)		128
Number of subcarriers (M)		96
Number of zero paddings (N_Z)		32
Convolutional FEC	Rate	1/2
	Constraint length	7
Conventional FDE [14]	Modulation for pilot	QPSK
	Pilot interval (FIP,TIP)	(4,1)
	Length of the guard interval (GI)	16
	Symbol duration (T_S)	108 μ s
Proposed TDE	Modulation for TS	16QAM
	Length of training sequence (TS)	16
	Symbol duration (T_S)	120 μ s
Modulation for data subcarrier		16QAM
Number of symbols per one frame (N_S)		33
Allocated bandwidth		1MHz
Radio frequency		5.9GHz
Rician multipath fading channel model		
Rician factor (K)		6dB
Delay profile		Exponential
Decay constant		-1dB
Number of delay paths (L)		14
Number of scattered rays		20

Table 3. Comparison of complexity ratios

C/N	Proposed iterative based TDE method ($N=128$, $TOL=0.01$)					
	$f_{dmax}/\Delta f = 10\%$		$f_{dmax}/\Delta f = 15\%$		$f_{dmax}/\Delta f = 20\%$	
	N_{Aver}	R	N_{Aver}	R	N_{Aver}	R
14dB	4.25	0.076	4.29	0.076	4.37	0.078
17dB	3.95	0.071	4.01	0.072	4.08	0.073
20dB	3.81	0.069	3.86	0.070	3.94	0.071

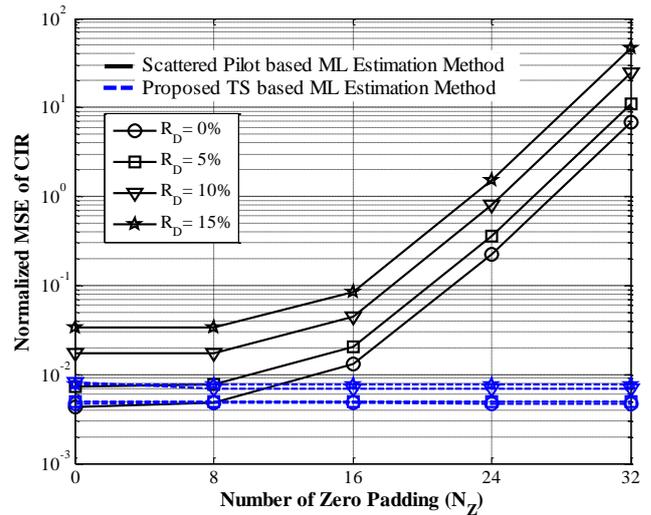


Figure. 8 CIR estimation accuracy (NMSE) for the proposed method at C/N=20dB

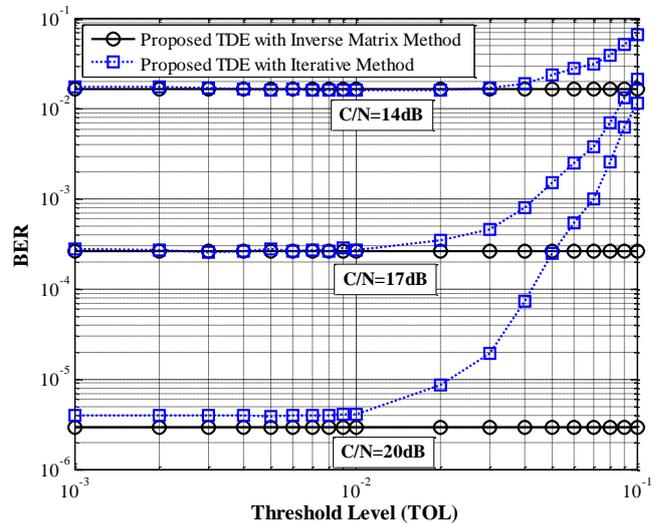


Figure. 9 BER performance for the proposed method when changing TOL at $R_D = 15\%$

Fig. 10 shows the average number of iterations for the proposed TDE with iterative method when changing R_D . In Fig. 10, it can be observed that the average number of iterations N_{Aver} becomes larger when increasing R_D and decreasing the operation C/N. By using the average number of iterations obtained in Fig. 10 and Eq. (45). Table 3 shows the ratio of computation complexity (R) between the proposed TDE with the iterative method and the inverse matrix method. In the Table 3, it can be concluded that the proposed TDE with the iterative method can reduce the computation complexity to 0.078 (Approximately 7.8%) at C/N is 14dB and R_D is 20% which corresponds to the vehicle speed 381 km/hrs.

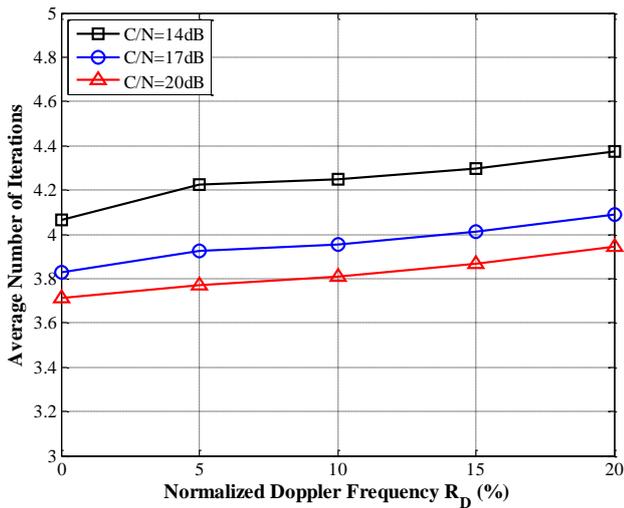


Figure. 10 An average number of required iterations for the proposed method when changing R_D at $TOL= 0.01$

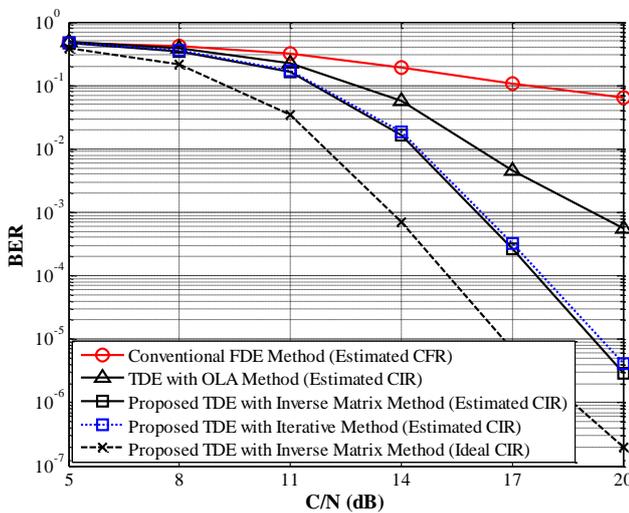


Figure. 11 BER performance for the proposed method when changing C/N at $R_D=15\%$

Fig. 11 shows the BER performances for the proposed TDE method both of the iterative and inverse matrix calculation methods when changing C/N at $R_D=15\%$ (Vehicle speed 286 km/hrs). In Fig. 11, it can be observed that the proposed TDE method can achieve much better BER performance than the conventional FDE method and the TDE with OLA method [13]. From the results in Fig. 11 and Table 3, it can be concluded that the proposed TDE with iterative method can achieve almost the same BER performance as the inverse matrix calculation method with much smaller computation complexity.

6. Conclusions

This paper proposes a low-complexity TDE method in conjunction with the time domain CIR estimation method. The salient features of the proposed method are to employ the time-domain

training sequence (TS) in the CIR estimation and to employ the partial differentiation in solving the ML equation. By using partial differentiation, the coefficients matrix of simultaneous equations becomes the square matrix which can be solved iteratively without the inverse matrix calculation. From the verified computer simulation results, it can be concluded that the proposed TDE with the iterative method can achieve much better BER performance than the conventional FDE and TDE with OLA methods under higher time-varying fading channels (vehicle speed ≈ 381 km/hrs). The computation complexity for the proposed TDE with iterative method shows much lower than the inverse matrix method up to 90% with the same BER performance of inverse matrix calculation.

In future work, the proposed channel estimation and equalization methods could be applied to multi-input multiple-output (MIMO) channels to increase the capacity by a factor of the minimum number of transmitting and receiving antennas.

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