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Integration of Group Method Analysis and Rough Sets Theory to Investigate Heat and Mass Transfer of the Flow of a Non-Newtonian Nanofluid towards a Vertical Stretching Surface

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Abstract: This work reports how to utilize rough set theory and group method analysis for generating a set of rules to investigate heat and mass transfer of mixed convection stagnation point flow of a non-Newtonian nanofluid towards a vertical stretching surface. By utilizing group method analysis, the main partial differential equations which describe the flow are rehabilitated to nonlinear Ordinary Differential Equations (ODEs). Then, the resultant nonlinear ODEs are solved numerically by applying the implicit finite-difference scheme. The numerical values thus obtained are depicted in tabular form and the basic principles of rough sets are applied to get all reducts and finally a group of generalized rules are extracted to predict the value of local Nusselt number and local skin-friction coefficient. The outcomes demonstrate the novelty of the current work which can be summarized as hybridization of group method analysis and rough set theory to use in the field of fluid dynamics effectively. The resultant set of generalized classification rules which performed with basic logic functions can be considered as knowledge base with high accuracy and may be valuable in many engineering applications like power production, thermal extrusion systems and microelectronics.

Keywords: Classification, Rules extraction, Group method analysis, Nanofluid, Thermophoresis, Local musselt number, Local skin-friction coefficient, Rough sets theory, Feature selection.

1. Introduction

The research of non-Newtonian fluids is curiosity to numerous investigators because non-Newtonian fluids exhibit a nonlinear linkage among the rate of strain and stresses. This linkage leads to sophisticated rheological attitude which is not experienced at treating ordinary low-molecular-weight Newtonian fluids, since its behavior cannot be qualified by a single constitutive relation. The flow of non-Newtonian fluids also gains more interest due to its emerging in different applications in natural sciences, engineering sciences and industry. Many studies on these fluids are investigated by many researchers [1–6].

Nanofluids became a theme with considerable interest during the final decade due to their ability to improve the heat transfer rate in engineering suits. Nanofluids have wide utilities in cars and manufacturing cooling, tumefaction therapy, cooling towers, age of modern type fuels, cool down of microelectronics, cleverness of cross breed fueled engines and warming/cooling of home instruments. The pioneer experimental results of the researchers Choi and Eastman [7] was the first to utilize the word nanoparticle by keeping in mind the end goal to expand the heat conductivity of base liquids, how the warm conductivity is expanded is the main perspective of their work. The role of Brownian motion, interfacial resistance, morphology of suspended nanoparticles and aggregating behavior is investigated both experimentally and theoretically [8, 9], Then various exploratory and numerical examinations have been exhibited to explain the thermal and physical properties on different fluid flow patterns and heat transfer problems. Buongiorno [10] considered seven slip mechanisms that can produce a relative velocity between the nanoparticles and the base fluid. These are inertia, Brownian diffusion, thermophoresis, diffusiophoresis, Magnus effect, fluid drainage, and gravity. The 3D MHD nonlinear radiative hybrid nanofluid flow across an irregular dimension sheet with slip effect is studied numerically by Iskander [11]. Tiwari [12] investigated numerically the behavior of nanofluids is inside a two-sided lid-driven differentially heated square cavity to gain insight into convective recirculation and flow processes induced by a nanofluid. Ferdows [13] investigates the magnetohydrodynamic flow of a dissipative nanofluid, including gyrotactic microorganisms along an exponentially moving sheet. For more flow patterns see [14–29].

Group method and their invariants offer a powerful, sophisticated, and methodical technique to obtain the group-invariant solutions called self-similarity transformation. Self-similarity transformations make a reduct of the independent variables numbers of a set of partial Differential equations (PDEs), leading to convert the non-linear governing PDEs into the ODEs. Analysis using Lie group has been executed by many scientists and applied by mathematicians in many investigations. The linear transformation group approach is developed by Bakier [30] to simulate problem of hydromagnetic heat transfer by mixed convection along vertical plate in a liquid saturated porous medium in the presence of melting and thermal radiation effects for opposing external flow. Nabwey [31] used Lie Group Analysis to identify the characteristics of unsteady magnetohydrodynamic (MHD) flow of ferrofluid past a radiated stretching surface. More studies can be found in [32-36].

Rough sets theory (RST) is one of the successful approximations based mathematical model to deal the imprecision and uncertainty present in knowledge [37]. Many heuristic algorithms are proposed based on rough set theory, also numerous approaches based on rough set theory and other theories are investigated to extract decision rules and reduce the dimensionality of dataset. Nabwey [38] proposes a hybrid approach based on rough set methodology and fuzzy inference system for extraction of classification rules. Also in [39] he introduced Rough set theory based method for building knowledge for the rate of heat transfer on free convection over a vertical flat plate embedded in a porous medium. An approach based on Rough Sets Theory and Grey System for Implementation of Rule-Based Control for Sustainability of Rotary Clinker Kiln was introduced in [40]. Shaaban et al. [41, 42] used rough set methodology for steam turbine-generator fault diagnosis and for water reservoirs site location decision making, respectively. For more applications see [43-54]. One advantage of the rough set is the creation of readable if-then rules. Such rules have a potential to reveal new patterns in the data material.
The main objective of this article is to inspect the effects of the Brownian motion and thermophoresis on mixed convection flow near stagnation point flow of a non-Newtonian nanofluid towards a vertical stretching surface, the model proposed by Buongiorno is employed for the nanofluid behavior. The basic governing partial Differential equations (PDEs) describing the problem are reduced into a similarity form using the one-parameter transformation group, and then the reduced similarity equations are solved numerically. Finally, the rough sets theory is applied to generate some classification rules to predict the different behaviors of relevant parameters on the local skin-friction coefficient and local Nusselt number. The proposed method put this work in a different position where we can obtain the values of local Nusselt number and local skin-friction coefficient at any values of the parameters under consideration and determine the most important parameters that affect the values of local Nusselt number and local skin-friction coefficient.

This paper is organized as follows: The Mathematical Framework of the problem is investigated in part two. Part three introduce the proposed methodology which is divided mainly two stages to extract the generalized decision rules. Then, the Outcomes and discussion are introduced and finally, concluded remarks are investigated.

2. The mathematical framework

According to [5] the mathematical model which describe the governing equations of steady, laminar, mixed convection, boundary layer flow of a non-Newtonian power-law nanofluid obeying the Ostwald-de Waele model [55] near stagnation-point at heated stretching vertical surface coinciding with the plane $y = 0$ with the following assumptions :
The flow being confined to the region $y = 0$.

The model proposed by Buongiorno is employed for the nanofluid behavior in which the effects of Brownian motion and thermophoresis are taken into consideration.

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$U_w(x) = ax$, where a is a constant
$U(x) = cx \; at \; x = y = 0$, where c is a constant

and under usual assumptions in the literature, it can be written as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (1)$$

$$\rho_{f\infty}\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \rho_{f\infty} U \frac{dU}{dx} + \frac{\partial \tau_{xy}}{\partial y} + (1 - C_\infty)\rho_{f\infty} g\beta(T - T_\infty) - (\rho_p - \rho_{f\infty})g(C - C_\infty) \qquad (2)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \tau\left[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y} + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right] \qquad (3)$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B\frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2} \qquad (4)$$

Subject to the following boundary conditions:

$$at \quad y = 0 \qquad u = U_w(x), \quad v = 0, \quad T = T_w, \quad D_B\frac{\partial C}{\partial y} + \frac{D_B}{T_\infty}\frac{\partial T}{\partial y} = 0 \qquad (5)$$

$$\text{as} \quad y \to \infty \quad u = U(x), \;\; T = T_\infty, \;\; C = C_\infty (6)$$

With the aid of the following non-dimensional parameters:

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}, \;\; \theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad \varphi = \frac{C - C_\infty}{C_\infty}, \quad Nr = \frac{(\rho_p - \rho_{f\infty})C_\infty}{\rho_{f\infty}\beta(T_w - T_\infty)(1 - C_\infty)} \quad (7)$$

The mathematical model represented by Eqs. (1) - (4) can be reduced to:

$$\frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial y^2} = a^2 x + \frac{K}{\rho_{f\infty}}\frac{\partial}{\partial y}\left(\frac{\partial^2 \psi}{\partial y^2}\right)^n + (1 - C_\infty)\rho_{f\infty} g\beta(T_w - T_\infty)(\theta - Nr\phi) \qquad (8)$$

$$\frac{\partial \psi}{\partial y}\frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial \theta}{\partial y} = \alpha\frac{\partial^2 \theta}{\partial y^2} + \tau\left[D_B C_\infty\frac{\partial \phi}{\partial y}\frac{\partial \theta}{\partial y} + \frac{D_T(T_w - T_\infty)}{T_\infty}\left(\frac{\partial \theta}{\partial y}\right)^2\right] \qquad (9)$$

$$\frac{\partial \psi}{\partial y}\frac{\partial \phi}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial \phi}{\partial y} = D_B\frac{\partial^2 \phi}{\partial y^2} + \frac{D_T(T_w - T_\infty)}{T_\infty C_\infty}\frac{\partial^2 \theta}{\partial y^2} (10)$$

Subject to the following boundary conditions:

$$at \; y = 0 \quad \frac{\partial \psi}{\partial y} = cx, \;\; \frac{\partial \psi}{\partial x} = 0, \;\; \theta = 1,$$

$$D_B C_\infty\frac{\partial \varphi}{\partial y} + \frac{D_B(T_w - T_\infty)}{T_\infty}\frac{\partial \theta}{\partial y} = 0 \qquad (11)$$

$$y \to \infty \quad \frac{\partial \psi}{\partial y} = ax, \;\; \theta = 0, \;\; \varphi = 0 \quad (12)$$

3. Proposed methodology

The proposed methodology consists of two main stages; the first stage is to use the one-parameter group to transform the mathematical model shown in (8) - (12) from PDE with two- independent variable to ODE in only one-independent variable While the second stage is to use the rough sets scheme to analyze, mining and generating generalized rules to predict the value of local Nusselt number, local skin-friction coefficient.

3.1 The first stage

This stage is initiated with the group G, a class of one-parameters 'b' of the form

$$\begin{aligned} \bar{x} &= C^x(b)x + K^x(b), \\ \bar{y} &= C^y(b)y + K^y(b), \\ \bar{\psi} &= C^\psi(b)\psi + K^\psi(b), \\ \bar{\theta} &= C^\theta(b)\theta + K^\theta(b), \\ \bar{\varphi} &= C^\varphi(b)\varphi + K^\varphi(b), \end{aligned} \qquad (13)$$

where Cs and Ks are real valued and at least differentiable in their real argument b and the transformations of the derivatives are obtained from *G* via chain-rule operations

$$\left.\begin{aligned} \bar{S}_{\bar{i}} &= (C^S/C^i)S_i \\ \bar{S}_{\bar{i}\bar{j}} &= (C^S/C^iC^j)S_{ij} \\ \bar{S}_{\bar{i}\bar{j}\bar{k}} &= (C^S/C^iC^jC^k)S_{ijk} \end{aligned}\right\} i,j,k = x,y \quad (14)$$

where *S* stands for ψ, θ, ϕ. Which summarize in a group *G* to transform invariantly the governing Eqs. (8) - (12) (for more Details see [5]) of the form:

$$G = \begin{cases} G_s = \begin{cases} \bar{x} = (C^y)^2 x, \\ \bar{y} = C^y y, \end{cases} \\ \begin{cases} \bar{\psi} = C^y\psi + K^\psi, \\ \bar{\theta} = \theta, \\ \bar{\varphi} = \varphi. \end{cases} \end{cases} \qquad (15)$$

The next step is to apply the basic theorem in group theory (see Moran and Gaggioli [56]) to find the complete set of absolute invariants which are the absolute invariants of the independent variables (x, y) is $\eta = \eta(x, y)$, and the absolute invariants of the dependent variables (ψ, θ, ϕ) are $\psi(x, y) =$

$\xi(x)F(\eta)$, $\theta(x,y)=\theta(\eta)$, $\phi(x,y)=\phi(\eta)$. Finally, reduce the governing equations to a set of ordinary differential equations in $F(\eta)$,$\theta(\eta)$ and $\phi(\eta)$ and hence transfer it to a set of ordinary differential equations in a single variable η by making the coefficients of the functions $F(\eta)$, $\theta(\eta)$, $\phi(\eta)$ and their derivatives, be an arbitrary constants or functions of η only. Yields the mathematical model:

$$n(F'')^{n-1}F''' + (\frac{2n}{n+1})FF'' - F'^2 + \frac{a^2}{c^2} + \Lambda(\theta - Nr\phi) = 0 \quad (16)$$

$$\theta'' + Pr\left(\frac{2n}{n+1}F\theta' + Nb\theta'\phi' + Nt\theta'^2\right) = 0 \quad (17)$$

$$\phi'' + Le\frac{2n}{n+1}F\phi' + \frac{Nt}{Nb}\theta'' = 0 \quad (18)$$

Subject to the following boundary conditions:

$$F(0)=0 \quad F'(0)=1 \quad \theta(0)=1$$
$$Nb\varphi'(0) + Nt\theta'(0) = 0 \quad (19)$$

$$F'(\infty) = \frac{a}{c}, \theta(\infty) = \phi(\infty) = 0 \quad (20)$$

Where

$$\lambda = \frac{Gr_x}{Re_x^2} \quad Pr = \frac{cx^2}{\alpha}Re_x^{-2/(n+1)} \quad Re_x = \frac{(cx)^{2-n}x^n}{K/\rho_{f\infty}}$$

$$Nb = \frac{\tau D_B C_\infty}{cx^2\, Re_x^{-\frac{2}{n+1}}} \quad Nt = \frac{\tau D_T(T_w - T_\infty)}{T_\infty cx^2\, Re_x^{-\frac{2}{n+1}}}$$
$$Le = \frac{cx^2}{D_B}Re_x^{-2/(n+1)} \quad Gr_x = \frac{(1-C_\infty)\rho_{f\infty}g\beta(T_w - T_\infty)}{(K/\rho_{f\infty})^2} \quad (21)$$

It is worth noting that the differentiation in (16) - (20) with respect to η and all the details can be found in [5].

The most significant parameters for this paper are the local skin friction C_f and local Nusselt number Nu_x, and these quantities can be written as:

$$C_f = 2\left(F''(0)\right)^n\left[\frac{(cx)^{2-n}x^n}{K/\rho_{f\infty}}\right]^{-1/(1+n)} \quad (22)$$

$$Nu_x = -\left(\frac{u_w^{2-n}}{K/\rho_{f\infty}}\right)^{1/(1+n)}\theta'(0) \quad (23)$$

Where the local skin friction factor Cf is proportional to the numerical values of (F"(0))n and and the local Nusselt number Nux is proportional to the numerical values of θ'(0).

Now it is required to obtain the solution of the Eqs. (16) – (18) subject to the boundary conditions in Eqs. (19) and (20). since formulated equations are highly nonlinear and coupled their analytical solutions are difficult, it is solved numerically with the aid of the

Table 1. Decision table of local skin friction coefficient for various values of Nr, Nb, Nt, n, Λ, a/c at Pr=6.8 and Le=10

U	Nr	Nb	Nt	n	Λ	*a/c*	local skin friction coefficient $(F''(0))^n$
X1	0.1	0.5	0.3	0.5	3.0	1.5	1.797508
X2	0.3	0.5	0.3	0.5	3.0	1.5	1.792772
X3	0.5	0.5	0.3	0.5	3.0	1.5	1.788013
X4	0.1	0.1	0.3	0.5	3.0	1.5	1.788109
X5	0.1	0.3	0.3	0.5	3.0	1.5	1.795914
X6	0.1	0.5	0.3	0.5	3.0	1.5	1.797508
X7	0.1	0.5	0.1	0.5	3.0	1.5	1.727497
X8	0.1	0.5	0.3	0.5	3.0	1.5	1.797508
X9	0.1	0.5	0.5	0.5	3.0	1.5	1.849623
X10	0.1	0.5	0.3	0.5	3.0	1.5	1.797508
X11	0.1	0.5	0.3	1.0	3.0	1.5	1.700608
X12	0.1	0.5	0.3	1.5	3.0	1.5	1.647145
X13	0.1	0.5	0.3	0.5	-2.0	1.5	0.399424
X14	0.1	0.5	0.3	0.5	1.0	1.5	1.352609
X15	0.1	0.5	0.3	0.5	2.0	1.5	1.587690
X16	0.1	0.5	0.3	0.5	3.0	1.5	1.797508
X17	0.1	0.5	0.3	0.5	3.0	1.7	2.052897
X18	0.1	0.5	0.3	0.5	3.0	1.9	2.301840

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Table 2. Decision table of local nusselt number for various values of Nr, Nb, Nt, n, Λ, a/c at Pr=6.8 and Le=10

U	Nr	Nb	Nt	n	Λ	*a/c*	local Nusselt number -θ'(0)
X1	0.1	0.5	0.3	0.5	3.0	1.5	0.825806
X2	0.3	0.5	0.3	0.5	3.0	1.5	0.824981
X3	0.5	0.5	0.3	0.5	3.0	1.5	0.824468
X4	0.1	0.1	0.3	0.5	3.0	1.5	0.824515
X5	0.1	0.3	0.3	0.5	3.0	1.5	0.825869
X6	0.1	0.5	0.3	0.5	3.0	1.5	0.825806
X7	0.1	0.5	0.1	0.5	3.0	1.5	1.391290
X8	0.1	0.5	0.3	0.5	3.0	1.5	0.825806
X9	0.1	0.5	0.5	0.5	3.0	1.5	0.554399
X10	0.1	0.5	0.3	0.5	3.0	1.5	0.825806
X11	0.1	0.5	0.3	1.0	3.0	1.5	0.975382
X12	0.1	0.5	0.3	1.5	3.0	1.5	1.056164
X13	0.1	0.5	0.3	0.5	-2.0	1.5	0.752429
X14	0.1	0.5	0.3	0.5	1.0	1.5	0.797467
X15	0.1	0.5	0.3	0.5	2.0	1.5	0.812213
X16	0.1	0.5	0.3	0.5	3.0	1.5	0.825806
X17	0.1	0.5	0.3	0.5	3.0	1.7	0.854231
X18	0.1	0.5	0.3	0.5	3.0	1.9	0.88326

implicit, tri-diagonal, finite-difference scheme. The numerical results are reported in Table 1 and Table 2 which represent the decision tables for local skin friction factor (F''(0))n, local Nusselt number -θ'(0), respectively [5].

3.2 The second stage

Rough sets schem in this stage, we will apply the basic principles of Rough sets theory to discover structural relationships within the given training data by establishment of equivalence classes then a definition for a given class is approximated by two sets, namely, a lower approximation of and an upper approximation as shown in Fig. 1 where each rectangular region represents an equivalence class. Finally, computation of reducts from data and derivation of rules are done.

The proposed rough sets methodology will be used to mining and generating generalized rules to predict the value of local skin friction coefficient and local Nusselt number for various values of Nr, Nb, Nt, n, Λ, a/c. the overall steps of this stage are shown in Fig. 2.

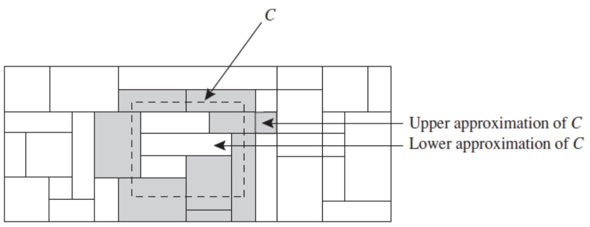

Figure. 1 Graphical representation of lower and upper approximation

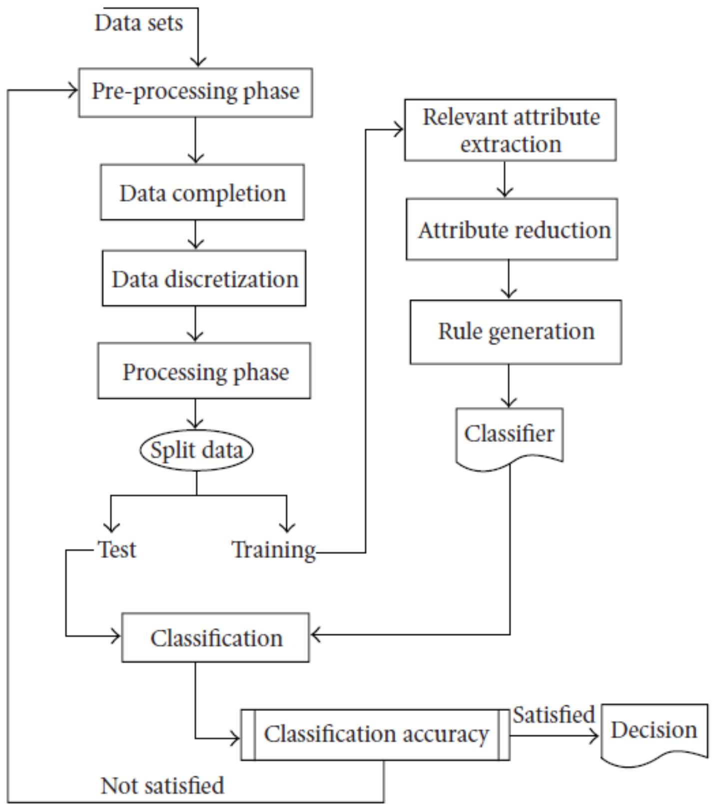

Figure. 2 The overall steps of the second stage

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4. Outcomes and discussion

By using software called ROSETTA which is an RST analysis toolkit, rough sets with Boolean reasoning discretization algorithm is introduced to discretize the data as shown in Table 3 and Table 4 where * means do not care condition. Next step is to find the equivalence classes as shown in Table 5, later the rough set reduction technique is applied to find the minimal subsets (reducts) of attributes that can describe all the concepts in the decision tables as shown in Table 6. Finally, the knowledge gained from all generated reducts can be represented by rough sets dependency rules as shown in Table 7 and Table 8.

Table 3. The discretized decision table of Table 1.

U	Nr	Nb	Nt	n	Λ	*a/c*	local skin friction coefficient $(F''(0))^n$
X1	(0.2 ,*]	(* ,0.4]	(0.4 ,0.2]	(0.8 ,*]	(* ,3]	(1.6 ,*]	1.79751
X2	(0.4 ,0.2]	(* ,0.4]	(0.4 ,0.2]	(0.8 ,*]	(* ,3]	(1.6 ,*]	1.79277
X3	(* ,0.4]	(* ,0.4]	(0.4 ,0.2]	(0.8 ,*]	(* ,3]	(1.6 ,*]	1.78801
X4	(0.2 ,*]	(0.2 ,*]	(0.4 ,0.2]	(0.8 ,*]	(* ,3]	(1.6 ,*]	1.78811
X5	(0.2 ,*]	(0.4 ,0.2]	(0.4 ,0.2]	(0.8 ,*]	(* ,3]	(1.6 ,*]	1.79591
X6	(0.2 ,*]	(* ,0.4]	(0.4 ,0.2]	(0.8 ,*]	(* ,3]	(1.6 ,*]	1.79751
X7	(0.2 ,*]	(* ,0.4]	(0.2 ,*]	(0.8 ,*]	(* ,3]	(1.6 ,*]	1.72750
X8	(0.2 ,*]	(* ,0.4]	(0.4 ,0.2]	(0.8 ,*]	(* ,3]	(1.6 ,*]	1.79751
X9	(0.2 ,*]	(* ,0.4]	(* ,0.4]	(0.8 ,*]	(* ,3]	(1.6 ,*]	1.84962
X10	(0.2 ,*]	(* ,0.4]	(0.4 ,0.2]	(0.8 ,*]	(* ,3]	(1.6 ,*]	1.79751
X11	(0.2 ,*]	(* ,0.4]	(0.4 ,0.2]	(1.3 ,0.8]	(* ,3]	(1.6 ,*]	1.70061
X12	(0.2 ,*]	(* ,0.4]	(0.4 ,0.2]	(* ,1.3]	(* ,3]	(1.6 ,*]	1.64715
X13	(0.2 ,*]	(* ,0.4]	(0.4 ,0.2]	(0.8 ,*]	(0 ,*]	(1.6 ,*]	0.39942
X14	(0.2 ,*]	(* ,0.4]	(0.4 ,0.2]	(0.8 ,*]	(2 ,0]	(1.6 ,*]	1.35261
X15	(0.2 ,*]	(* ,0.4]	(0.4 ,0.2]	(0.8 ,*]	(3 ,2]	(1.6 ,*]	1.58769
X16	(0.2 ,*]	(* ,0.4]	(0.4 ,0.2]	(0.8 ,*]	(* ,3]	(1.6 ,*]	1.79751
X17	(0.2 ,*]	(* ,0.4]	(0.4 ,0.2]	(0.8 ,*]	(* ,3]	(1.8 ,1.6]	2.05290
X18	(0.2 ,*]	(* ,0.4]	(0.4 ,0.2]	(0.8 ,*]	(* ,3]	(* ,1.8]	2.30184

Table 4. The discretized decision table of Table 2

U	Nr	Nb	Nt	n	Λ	*a/c*	local Nusselt number $-\theta'(0)$
X1	(0.2 ,*]	(* ,0.4]	(0.4 ,0.2]	(0.8 ,*]	(* ,3]	(1.6 ,*]	1.79751
X2	(0.4 ,0.2]	(* ,0.4]	(0.4 ,0.2]	(0.8 ,*]	(* ,3]	(1.6 ,*]	1.79277
X3	(* ,0.4]	(* ,0.4]	(0.4 ,0.2]	(0.8 ,*]	(* ,3]	(1.6 ,*]	1.78801
X4	(0.2 ,*]	(0.2 ,*]	(0.4 ,0.2]	(0.8 ,*]	(* ,3]	(1.6 ,*]	1.78811
X5	(0.2 ,*]	(0.4 ,0.2]	(0.4 ,0.2]	(0.8 ,*]	(* ,3]	(1.6 ,*]	1.79591
X6	(0.2 ,*]	(* ,0.4]	(0.4 ,0.2]	(0.8 ,*]	(* ,3]	(1.6 ,*]	1.79751
X7	(0.2 ,*]	(* ,0.4]	(0.2 ,*]	(0.8 ,*]	(* ,3]	(1.6 ,*]	1.72750
X8	(0.2 ,*]	(* ,0.4]	(0.4 ,0.2]	(0.8 ,*]	(* ,3]	(1.6 ,*]	1.79751
X9	(0.2 ,*]	(* ,0.4]	(* ,0.4]	(0.8 ,*]	(* ,3]	(1.6 ,*]	1.84962
X10	(0.2 ,*]	(* ,0.4]	(0.4 ,0.2]	(0.8 ,*]	(* ,3]	(1.6 ,*]	1.79751
X11	(0.2 ,*]	(* ,0.4]	(0.4 ,0.2]	(1.3 ,0.8]	(* ,3]	(1.6 ,*]	1.70061
X12	(0.2 ,*]	(* ,0.4]	(0.4 ,0.2]	(* ,1.3]	(* ,3]	(1.6 ,*]	1.64715
X13	(0.2 ,*]	(* ,0.4]	(0.4 ,0.2]	(0.8 ,*]	(0 ,*]	(1.6 ,*]	0.39942
X14	(0.2 ,*]	(* ,0.4]	(0.4 ,0.2]	(0.8 ,*]	(2 ,0]	(1.6 ,*]	1.35261
X15	(0.2 ,*]	(* ,0.4]	(0.4 ,0.2]	(0.8 ,*]	(3 ,2]	(1.6 ,*]	1.58769
X16	(0.2 ,*]	(* ,0.4]	(0.4 ,0.2]	(0.8 ,*]	(* ,3]	(1.6 ,*]	1.79751
X17	(0.2 ,*]	(* ,0.4]	(0.4 ,0.2]	(0.8 ,*]	(* ,3]	(1.8 ,1.6]	2.05290
X18	(0.2 ,*]	(* ,0.4]	(0.4 ,0.2]	(0.8 ,*]	(* ,3]	(* ,1.8]	2.30184

Table 5. The equivalence classes of discretized decision table

Eq. Classes	{4}	{5}	{7}	{13}	{14}	{15}	{1, 6, 8, 10, 16}	{17}
Cardinality	1	1	1	1	1	1	5	5
Eq. Classes	{18}	{11}	{12}	{9}	{2}	{3}	{11}	
Cardinality	1	1	1	1			1	

Table 6. Reducts of discretized decision table

Reduct	{ Nr, Nb, Nt, n, Λ, a/c }	{ Nr}	{ Nb }	{ Nt }	{n}	{Λ}	{ a/c }
Support	100	100	100	100	100	100	100
Length	6	1	1	1	1	1	1

Table 7. The generated rules to predict the value of local skin friction coefficient $(F''(0))^n$

	Rule	LHS SUPPORT	RHS SUPPORT	RHS ACCURACY	LHS COVERAGE
1	If Nr ([*, 0.2)) AND Nb ([0.4, *)) AND Nt ([0.2, 0.4)) AND , n ([*, 0.8)) AND Λ ([3, *)) AND a/c ([*, 1.6)) => $(F''(0))^n$ (1.79751)	5	5	1.0	0.277778
2	If Nr ([0.2, 0.4)) => $(F''(0))^n$ (1.79277)	1	1	1.0	0.055556
3	If Nr ([0.4, *)) => $(F''(0))^n$ (1.78801)	1	1	1.0	0.055556
4	If Nb ([*, 0.2)) => $(F''(0))^n$ (1.78811)	1	1	1.0	0.055556
5	If Nb ([0.2, 0.4)) => $(F''(0))^n$ (1.79591)	1	1	1.0	0.055556
6	If Nt ([*, 0.2)) => $(F''(0))^n$ (1.72750)	1	1	1.0	0.055556
7	If Nt ([0.4, *)) => $(F''(0))^n$ (1.84962)	1	1	1.0	0.055556
8	If n ([0.8, 1.3)) => $(F''(0))^n$ (1.70061)	1	1	1.0	0.055556
9	If n ([1.3, *)) => $(F''(0))^n$ (1.64715)	1	1	1.0	0.055556
10	If Λ ([*, 0)) => $(F''(0))^n$ (0.39942)	1	1	1.0	0.055556
11	If Λ ([0, 2)) => $(F''(0))^n$ (1.35261)	1	1	1.0	0.055556
12	If Λ ([2, 3)) => $(F''(0))^n$ (1.58769)	1	1	1.0	0.055556
13	If a/c ([1.6, 1.8)) => $(F''(0))^n$ (2.05290)	1	1	1.0	0.055556
14	If a/c ([1.8, *)) => $(F''(0))^n$ (2.30184)	1	1	1.0	0.055556

As shown in Tables 7 and 8, the extracted decision rules represent the impact of some thermophysical parameters such as thermophoresis parameter, power-law index parameter, ratio of velocity parameter, Brownian motion parameter, nanoparticle buoyancy ratio and mixed convection parameter on the local skin friction coefficient $(F''(0))^n$ and local Nusselt number $-\theta'(0)$. It is found that as the power-index increases there is an increase in $-\theta'(0)$ and decrease in $(F''(0))^n$ due to the reduction of the thermal boundary-layer thickness which leads to decreasing in the nanofluid velocity, this produce a depress in the wall shear stress and improvement of rate of heat transfer at the surface. as well as the increase in the velocity ratio and mixed convection parameters result in accelerating the flow which in turn cause a reduction in the fluid temperature and increases the maximum velocity leading to the increase of $(F''(0))^n$ and $-\theta'(0)$. It is also noted that a slight reduction in $(F''(0))^n$ and $-\theta'(0)$ is occurred due to the increase in nanoparticle buoyancy ratio. Moreover a slight enhancement in $(F''(0))^n$ and $-\theta'(0)$ is occurred as the Brownian parameter increased because Brownian diffusion increase heat conduction. In order to check the effectiveness of the proposed method and test the accuracy of the obtained results, the results for skin friction coefficient $F''(0)$ are compared with those of Wang [57], Gorla and Sidawi [58] and Nabwey et al. [5] for different values of the related parameters as shown in Table 9. From these comparisons it is clear that there is an excellent agreement and the accuracy of the proposed method is high.

This work introduced a methodology based on group method analysis and rough sets theory for generating a set of decision rules to investigate and predict the value of local Nusselt number and local skin-friction coefficient considering the effects of the Brownian motion and thermophoresis on mixed convection stagnation point flow of a Non-Newtonian

Table 8.The generated rules to predict the value of local nusselt number -θ'(0)

	Rule	LHS SUPPORT	RHS SUPPORT	RHS ACCURACY	LHS COVERAGE
1	If Nr ([*, 0.2)) AND Nb ([0.4, *)) AND Nt ([0.2, 0.4)) AND , n ([*, 0.8)) AND Λ ([3, *)) AND a/c ([*, 1.6)) => -θ'(0) (0.82581)	5	5	1.0	0.277778
2	If Nr ([0.2, 0.4)) => -θ'(0) (0.82498)	1	1	1.0	0.055556
3	If Nr ([0.4, *)) => -θ'(0) (0.82447)	1	1	1.0	0.055556
4	If Nb ([*, 0.2)) => -θ'(0) (0.82451)	1	1	1.0	0.055556
5	If Nb ([0.2, 0.4)) => -θ'(0) (0.82587)	1	1	1.0	0.055556
6	If Nt ([*, 0.2)) => -θ'(0) (1.39129)	1	1	1.0	0.055556
7	If Nt ([0.4, *)) => -θ'(0) (0.55440)	1	1	1.0	0.055556
8	If n ([0.8, 1.3)) => -θ'(0) (0.97538)	1	1	1.0	0.055556
9	If n ([1.3, *)) => -θ'(0) (1.05616)	1	1	1.0	0.055556
10	If Λ ([*, 0)) => -θ'(0) (0.75243)	1	1	1.0	0.055556
11	If Λ ([0, 2)) => -θ'(0) (0.79747)	1	1	1.0	0.055556
12	If Λ ([2, 3)) => -θ'(0) (0.81221)	1	1	1.0	0.055556
13	If a/c ([1.6, 1.8)) => -θ'(0) (0.85423)	1	1	1.0	0.055556
14	If a/c ([1.8, *)) => -θ'(0) (0.88326)	1	1	1.0	0.055556

Table 9. Comparison of $F''(0)$ for various values of Pr at n=1, Λ=Nr=0

U	Pr	Wang [57]	Gorla and Sidawi [58]	Nabwey et al. [5]	Current work
X1	0.07	0.0656	0.0656	0.06629	0.0660
X2	0.2	0.1691	0.1691	0.16913	0.1692
X3	0.7	0.4539	0.5349	0.45394	0.4545
X4	2.0	0.9113	0.9113	0.91132	0.9199
X5	7.0	1.8954	1.8905	1.89545	1.9
X6	20.0	3.3539	3.3539	3.35389	3.3563
X7	70.0	6.4622	6.4622	6.46221	6.4444

nanofluid towards a vertical stretching surface. It is observed that the skin friction coefficient reduces due to the increase in either of the nanoparticle buoyancy ratio or power-law fluid index whereas the effect reverses for the Brownian motion parameter, thermophoresis parameter, mixed convection parameter and velocity ratio. Furthermore, it is found that the increase in the Brownian motion parameter have enhancing effects on the local Nusselt number, while the increase in the thermophoresis parameter falls significantly the Nusselt number. Also, increases in the values of either of the velocity ratio or the power-law fluid index produced enhances in the local Nusselt number whereas, the opposite behavior was predicted as the mixed convection parameter and nanoparticle buoyancy ratio were increased for which the local Nusselt number was depressed. These results demonstrate the novelty of the current work which can be summarized as hybridization of group method analysis and rough set theory to use in the field of fluid dynamics effectively. The proposed technique has simplified logic-based rules required to effectively predict these values with high accuracy and may be valuable in many engineering applications and be considered as knowledge base.

5. Conclusion

An extension work of using rough sets with other intelligent systems like neural networks, genetic algorithms, fuzzy approaches, and so forth, will be considered in the future work.

Conflicts of Interest

The author declare no conflict of interest

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Nomenclature

x and y	The Cartesian Coordinates
$U_w(x)=ax$	the stretching velocity
$U(x)=cx$	the velocity of external flow
T_∞	the ambient temperature far away from the surface
C_∞	The nanoparticle volume fraction far away from the surface
C	nanoparticle volume fraction
(u,v)	the velocity components along the (x,y)
T	the fluid temperature
β	the volumetric expansion coefficient of the fluid
g	gravitational acceleration
υ	kinematic viscosity
k	thermal conductivity
ρ_f	fluid density
ρ_p	nanoparticles mass density
α	thermal diffusivity of the nanofluid
C_f	the local skin friction
Nu_x	local Nusselt number
θ	Dimensionless temperature
ϕ	Dimensionless nanoparticle volume fraction
ψ	Stream function
$(\rho c)_f$	the heat capacity of the fluid
$(\rho c)_p$	the effective heat capacity of the nanoparticle
D_B	the Brownian diffusion coefficient
D_T	the thermophoretic diffusion coefficient
K	consistency coefficient
n	the power-law fluid.
λ	the dimensionless mixed convection parameter
Gr_x	the local Grashof number
Re_x	the local Reynolds number
Nb	the Brownian motion parameter
Nt	the thermophoresis parameter
Pr	the Prandtl number
Le	the Lewis number.
a/c	ratio of velocity parameter
Λ	the mixed convection parameter
Nr	buoyancy ratio