Energy Distribution and Loss Mitigations in DC Mass Rapid Transit by Using On-board Supercapacitor

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Abstract: In general, the train has consumed energy from the traction substation via the conductor during acceleration or motoring mode. This mode consumes most of the energy. However, during deceleration or braking mode, energy can be returned to the power terminal by regenerative braking management. This paper presents the energy loss reduction in the conductor transmission system and energy distribution mitigation from the traction substations by using the on-board supercapacitor, which is a storage device for storing regenerative braking energy. The results include the first case without a supercapacitor and the second case with supercapacitor operated on piece-wise linear state of charge (SOC) control. They are obtained by the single-train simulation on MATLAB and the power flow calculation by current injection method (CIM) on the peak time of the DC Mass Rapid Transit (MRT) Purple Line in Bangkok, Thailand. This research can effectively save energy distribution, energy loss and energy waste by approximately 3.5 %, 10 % and 20 % respectively.

Keywords: On-board energy storage, Piece-wise linear, Regenerative energy management, Single-train simulation, Supercapacitor.

Notation list

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_w$</td>
<td>is the passenger factor,</td>
</tr>
<tr>
<td>$A, B, C$</td>
<td>are the Davis coefficients,</td>
</tr>
<tr>
<td>$a$</td>
<td>is the acceleration,</td>
</tr>
<tr>
<td>$E_{\text{sub}}$</td>
<td>is the energy of traction substation,</td>
</tr>
<tr>
<td>$E_{\text{train}}$</td>
<td>is the train’s consumption energy,</td>
</tr>
<tr>
<td>$E_{\text{loss}}$</td>
<td>is the energy loss in conductor,</td>
</tr>
<tr>
<td>$E_{\text{waste}}$</td>
<td>is the energy waste,</td>
</tr>
<tr>
<td>$E_{\text{discharge}}$</td>
<td>is the energy discharge of energy storage device,</td>
</tr>
<tr>
<td>$E_{\text{charge}}$</td>
<td>is the energy charge of energy storage device,</td>
</tr>
<tr>
<td>$F$</td>
<td>is the force action on the train,</td>
</tr>
<tr>
<td>$F_{\text{friction}}$</td>
<td>is the train’s friction force,</td>
</tr>
<tr>
<td>$F_R$</td>
<td>is the train’s resistance force between wheel, rail and air,</td>
</tr>
<tr>
<td>$F_{\text{grad}}$</td>
<td>is the resistance force by gradient,</td>
</tr>
<tr>
<td>$g$</td>
<td>is the gravitational,</td>
</tr>
<tr>
<td>$G_k,j_i$</td>
<td>is the conductance matrix $k$ at bus $j$ to $i$,</td>
</tr>
<tr>
<td>$[G]$</td>
<td>is the matrix of conductor in the system,</td>
</tr>
<tr>
<td>$\Delta h$</td>
<td>is the difference of route elevation,</td>
</tr>
<tr>
<td>$I_{sx,j}$</td>
<td>is the short-circuit capacity of the traction substation at bus $j$,</td>
</tr>
<tr>
<td>$[I]$</td>
<td>is the vector of current in the system,</td>
</tr>
<tr>
<td>$I_{sc}$</td>
<td>is the current of supercapacitor,</td>
</tr>
<tr>
<td>$I_{sc,max}$</td>
<td>is the maximum current of supercapacitor,</td>
</tr>
<tr>
<td>$L$</td>
<td>is the route’s length,</td>
</tr>
<tr>
<td>$M_{\text{eff}}$</td>
<td>is the train’s mass effectiveness,</td>
</tr>
<tr>
<td>$M_t, M_l$</td>
<td>are the train’ mass and train’s payload mass,</td>
</tr>
<tr>
<td>$Md, Mc$</td>
<td>are the slope of discharge and charge,</td>
</tr>
<tr>
<td>$n$</td>
<td>is the number of cars,</td>
</tr>
<tr>
<td>$NS$</td>
<td>is the number of traction substations,</td>
</tr>
<tr>
<td>$NT$</td>
<td>is the simulation time with updating every 0.5 seconds,</td>
</tr>
<tr>
<td>$NP$</td>
<td>is the number of the next passenger station,</td>
</tr>
<tr>
<td>$OF$</td>
<td>is the objective function,</td>
</tr>
<tr>
<td>$P_{\text{train}}$</td>
<td>is the power consumed by the train,</td>
</tr>
<tr>
<td>$P_{\text{sub}}$</td>
<td>is the power of traction substation,</td>
</tr>
<tr>
<td>$P_{\text{loss}}$</td>
<td>is the power loss,</td>
</tr>
<tr>
<td>$P_{sc}$</td>
<td>is the power of supercapacitor,</td>
</tr>
<tr>
<td>$s$</td>
<td>is the position train of round-trip service,</td>
</tr>
<tr>
<td>$SNP$</td>
<td>is the position of passenger station number $NP$.</td>
</tr>
</tbody>
</table>
\(SOC_{end}\) is the state of charge at the end of round trip, 
\(TE\) is the train’s tractive effort, 
\(t, \Delta t\) are the simulation time and time step, 
\(v\) is the speed of a train,  
\(V_{\text{train}}\) is the train’s voltage,  
\(V_i\) is the voltage at bus \(i\),  
\([V]\) is the vector of voltage in the system,  
\(V_{sc}\) is the voltage of supercapacitor,  
\(\Delta V\) is the voltage deviation between train and supercapacitor,  
\(\Delta V_{d,\text{min}}\) is the minimum discharge voltage deviation,  
\(\Delta V_{d,\text{max}}\) is the maximum discharge voltage deviation,  
\(\Delta V_{c,\text{min}}\) is the minimum charge voltage deviation,  
\(\Delta V_{c,\text{max}}\) is the maximum charge voltage deviation,  
\(v_{\text{limit},NP}\) is the speed operation limit of passenger station number \(NP\),  
\(W_m, W_t\) are the motor car mass and trailer car mass.

1. Introduction

The worldwide demand for public transportation is increasing steadily in modern-day. Particularly in Thailand, where there are plans to extend the existing public transit system [1]. Generally, the electric train has three operating modes consist of motoring or acceleration mode, constant speed mode, and braking or deceleration mode. The mass rapid transit is characterized by frequently motoring and braking, which results in high regeneration power. The most energy is consumed by the train motoring and the energy can be regenerated by the train braking. Though regeneration energy is a waste product that must be removed from the system. An on-board supercapacitor deployed on a train is a viable solution for storing the regenerative braking energy during deceleration mode and supplying energy back to the traction system during acceleration mode. [2-4]. Therefore, the main advantages are the saving of energy consumption, energy loss and energy waste from braking reductions. There is a lot of paper involved in the management of regenerative braking energy, for example, determining the optimal sizing of an on-board energy storage device with a focus on reducing \(CO_2\) emissions or determining the optimal installation location of an energy storage device [5-6]. According to the results of the paper, the installation of wayside energy storage devices can reduce energy consumption. However, there is a limitation on power loss in transmission lines. Therefore, this paper focuses on the purpose of energy saving and loss minimization in MRT Purple Line in Bangkok, Thailand, by controlling an on-board supercapacitor to reuse the regenerative braking with a novel approach in applying the simulation model of the piece-wise linear SOC.

The train’s movement is calculated using a single-train simulation, and the power flow is calculated by using the current injection method (CIM) calculated by MATLAB [7], which calculates the train voltage, current, and power using a model of the multi-conductor. The electrochemical double-layer capacitor (EDLCs), also known as supercapacitors, have negligible chemical reactions at the electrodes, offers very low internal resistance, and thus has a very high efficiency of around 95%, making it the most ideal for the train’s power requirements both in accelerating and braking modes. The main advantage of a supercapacitor is faster charge/discharge than batteries, and higher efficiency [8-11]. Based on the stationary flywheel used in the New York City transportation with DC system and the technology for recharging and discharging energy storage, the goal of this regulation is to lower peak power demand in the power supply while simultaneously saving energy through regenerative braking. Technology has advanced significantly in recent years. On-board control state of charge (SOC) is becoming increasingly popular. On-board state of charge control for storage devices in DC systems has many ways of management [2, 12]. The notion of piece-wise linear state of charge control is investigated in this paper. It is a discharge of the current if the train voltage is lower than the energy storage device. If the train voltage is more, it is charged current. The variation of the charge and discharge current is slightly controlled by the linear equation. The charge and discharge area are developed from current regulation control method [13]. The five-zones piece-wise linear divide the working principle. There are two active energy storage zones and three non-active energy storage zones. There is a discrepancy in train voltage during operation. With an active slope, the current has been charging and discharging.

Consequently, this paper provides a technique for optimizing the best deviation of energy storage voltage for minimizing energy distribution coordination with considering the different voltages between train and energy storage. This problem was solved using the particle swarm optimization (PSO) approach. The initial energy can be any value, but the final energy storage must be full in 100% of SOC, and the voltage deviation must be in the device’s lowest to maximum range.

In this paper, section 2 presents the methodology for calculation including train movement, power flow analysis and piece-wise linear state of charge control. Section 3 proposes the concept of particle swarm optimization and optimal energy distribution in a DC railway system, as well as the problem formulation.
Section 4 describes the procedure of simulation and system parameters. Section 5 and 6 include the simulation results and conclusion, respectively.

2. Methodology

2.1 Train movement

Fig. 1 shows a free-body diagram of a train motion, which can be represented as the force acting on the train. The tractive effort and the friction forces operate on a train’s movement, as in Eq. (1). As seen in the diagram, the friction forces are comprised of the train resistance and the gradient force as in Eq. (2).

\[ F = TE - F_{friction} \]  
\[ F_{friction} = F_R + F_{grad} \]  

The train resistance, commonly known as the Davis equation, is the friction that occurs between the wheel, rail, and air while the train moving as in Eq. (3). The Davis equation was created particularly for each train model, and it was utilized in Eq. (4) by the JIS E 6002 Japan standard [14].

\[ F_R = A + Bv + Cv^2 \]  
\[ F_R = (1.65 + 0.0247v(t))W_m + (0.78 + 0.0028v(t))W_l + 0.0078(n - 1)v(t)^2 \]  

The gradient or slope of the metro rail line is frequently quite small; the force induced by the slope’s grade is computed as in Eq. (5).

\[ F_{grad} = M_{eff}gsin\theta \approx \frac{M_{eff}\Delta h}{L} \]  
\[ M_{eff} = M_t(1 + \lambda_w) + M_I \]  

The movement of a train can be calculated and modified every time steps of the simulations as in Eqs. (7) and (8). The train has three operational modes throughout the route between passenger stations including motoring or acceleration mode, constant speed mode, and braking or deceleration mode. The train accelerates from the passenger station in constant acceleration. When the train gets the operation limited speed, the mode switches to constant speed until the train starts braking at the next passenger station.

\[ v(t + \Delta t) = v(t) + a\Delta t \]  
\[ s(t + \Delta t) = s(t) + v(t + \Delta t)\Delta t + \frac{1}{2}a\Delta t^2 \]  

The train’s power is composed of auxiliary power for illuminating, conditioning systems, and communication, as well as traction power for driving the train, which is determined by traction force and instantaneous speed. The power of a train in deceleration or braking mode can return to storage in the on-board energy storage as expressed in Eq. (9). The train’s speed profile includes the train’s running duration, dwell time, and distance between passenger stations.

\[ P_{train} = \begin{cases} 
\frac{Fv(t+\Delta t)}{\eta} + P_{aux} & , a > 0 \\
P_{aux} & , a = 0 \\
Fv(t + \Delta)\eta + P_{aux} & , a < 0 
\end{cases} \]  

The power consumed by the train has been adjusted by on-board energy storage in order to calculate the energy traction substation has to supply as calculated in the following section.

2.2 Power flow calculation

The current injection method is generally regarded as a tool for solving electrical power networks in DC railways [1, 14, 15]. It is utilized in power flow calculations that is considered the multi-
conductor and single-train systems. The rectifier traction substation is modeled by Norton’s equivalent circuit as shown in Fig. 2, which the solution is calculated in Eq. (10) and Eq. (11).

\[ I_{ss,j} + \frac{P_{train}}{V_{train}} = \sum_{k=1}^{NS} G_{k,j} V_i \]  
\[ [I] = [G][V] \]  

The bus conductance matrix G in DC system can be used to express the bus admittance matrix Y in AC system. The matrix G consists of the resistance of conductors and rail.

The energy conservation law can describe the total energy in the based case as in Eq. (12), in which the braking power of the train is disposed of the waste energy expressed in Eq. (13). The distribution energy has some power supplied from on-board device in the case study calculated in Eq. (14). In this case some of the regenerative energy is capable to store in the on-board energy storage device as in Eq. (15). The waste energy will be reduced by stored in energy storage.

**Based case:** Without on-board energy storage,

\[ \sum_{i=1}^{NS} E_{sub,i} = E_{train} + \sum_{i=1}^{NS} E_{loss,i} \]  
(12)

\[ E_{waste} = E_{train}, a < 0 \]  
(13)

**Case study:** With on-board energy storage,

\[ \sum_{i=1}^{NS} E_{sub,i} = (E_{train} - E_{discarge}) + \sum_{i=1}^{NS} E_{loss,i} \]  
(14)

\[ E_{waste} = E_{train} - E_{charge}, a < 0 \]  
(15)

### 2.3 Piece-wise linear state of charge (SOC) control

The state of charge control in this paper, as a relationship between the voltage deviation of the on-board and the current of the energy storage shown in Fig. 3.

The difference between the train’s voltage and energy storage voltage is the voltage deviation as in Eq. (16). A piece-wise linear function with five zones has this property. Active on-board energy storage is divided into two zones, whereas non-active energy storage is divided into three zones. The active zone is defined by the voltage deviation's minimum and maximum values. As a result, the on-board energy storage is activated when the voltage deviation is positive exceeding the minimum discharge voltage deviation. The on-board energy storage current increases linearly with slope \( Md \) larger value of voltage deviation until the device’s maximum current is reached as in Eqs. (17-19). When the voltage deviation is negative and exceeding the minimum charge voltage deviation, the energy storage has been recharged. Until the device’s maximum current is reached, the current increases linearly with slope \( Mc \) as shown in Eqs. (20-22). When the current is not in the above conditions, the on-board device will not work as in Eq. (23) [13].

\[ \Delta V = |V_{train} - V_{sc}| \]  
(16)

The characteristic can be described as follows.

**Discharging zone:** the current is positive,

If \( \Delta V_{d,min} < \Delta V < \Delta V_{d,max} \)

\[ I_{sc} = M_d (\Delta V - \Delta V_{d,min}) \]  
(17)

\[ M_d = \frac{l_{sc,max}}{(\Delta V_{d,max} - \Delta V_{d,min})} \]  
(18)

else if \( \Delta V \geq \Delta V_{d,max} \)

\[ I_{sc} = I_{sc,max} \]  
(19)

**Charging zone:** the current is negative,

If \( \Delta V_{c,min} < \Delta V < \Delta V_{c,max} \)

\[ I_{sc} = M_c (\Delta V - \Delta V_{c,min}) \]  
(20)

\[ M_c = \frac{l_{sc,max}}{(\Delta V_{c,max} - \Delta V_{c,min})} \]  
(21)

else if \( \Delta V \geq \Delta V_{c,max} \)

\[ I_{sc} = I_{sc,max} \]  
(22)

Figure 3 The characteristic of the piece-wise linear state of charge control

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3. Problem formulation

3.1 Optimization problem

When the train leaves the passenger station, it must use a large amount of power from the traction substation powering it’s to achieve the operating speed. The power supplied via the conductor rail to the train has some loss depending on the distance between the traction substation and the train. If the train is located far from the traction substation and uses power for accelerating speed, the traction substation is heavily burdened to supply power. This paper studies about reducing energy distribution from the traction substation by installing an on-board supercapacitor and operating by state of charge control with the piece-wise linear method. The objective function can be expressed as follows:

\[
\text{Minimize: } OF = \sum_{i=1}^{NS} E_{\text{sub},i} \tag{24}
\]

Where,

\[
\sum_{i=1}^{NS} E_{\text{sub},i} = \sum_{d_{t}=0.5}^{NT} \sum_{i=1}^{NS} (P_{\text{sub},d_{t},i}) \tag{25}
\]

\[
\sum_{d_{t}=0.5}^{NT} \sum_{i=1}^{NS} P_{\text{sub},d_{t},i} = \sum_{d_{t}=0.5}^{NT} ((P_{\text{train},d_{t}} + P_{\text{sc},d_{t}}) + \sum_{i=1}^{NS} P_{\text{loss},d_{t},i}) \tag{26}
\]

\[
P_{\text{sc}} = I_{\text{sc}} V_{\text{sc}} \tag{27}
\]

\[
I_{\text{sc}} = \begin{cases} 
M(c_{\text{ΔV}} - c_{\text{ΔV}_{\text{c,min}}}) , & \text{ΔV}_{\text{c,min}} < \Delta V < \Delta V_{\text{c,max}} \\
M(d_{\text{ΔV}} - d_{\text{ΔV}_{\text{d,min}}}) , & \text{ΔV}_{\text{d,min}} < \Delta V < \Delta V_{\text{d,max}} \\
0 , & \text{ΔV}_{\text{c,min}} < \Delta V < \Delta V_{\text{d,min}}
\end{cases} \tag{28}
\]

Subject to the constraint,

\[
SOC_{\text{end}} = 100 \tag{29}
\]

The coordination constraint,

\[
0 < \Delta V < \Delta V_{\text{d,max}} \tag{30}
\]

The operation train constraint,

\[
s \geq s_{NP} \tag{31}
\]

Non-activated zone: the current is zero,

If \( \Delta V_{\text{c,min}} < \Delta V < \Delta V_{\text{d,min}} \)

\[
I_{\text{sc}} = 0 \tag{23}
\]

3.2 Particle swarm optimization technique

Particle swarm optimization (PSO) technique is a well-known method of optimization. One of the strategies for achieving results quickly is to use the work of particles in the dispersion to identify the best value [16-18]. The PSO approach is utilized in this paper to optimize the minimum discharge deviation voltage to manage the energy storage operation, which uses on-board energy storage to reduce the train’s power consumption. The proposed technique computation is shown step by step in Fig. 4.

\[
v = v_{\text{lim}it, NP} \tag{32}
\]

**Step 1:** Random initial position of the population matrix \((X)\) within the limited boundary in Eq. (30) with 50 swarm size.

\[
X = [\Delta V_{\text{d,mi}}, \ldots, \Delta V_{\text{d,mi}}] \tag{33}
\]

**Step 2:** Calculate the train movement and power flow to achieve the power consumption, which the power consumption has been adjusted by power...
discharge from energy storage by substitute $X$ from Eq. (33) in Eqs. (26-28).

**Step 3:** Repeat step 2 with an update time step every 0.5 seconds, checking the train’s position using the constraint in Eq. (31).

**Step 4:** Solve the objective function from Eq. (24) subject to the constraint in Eqs. (29-32).

**Step 5:** Find the $p_{best}$ that is the particle best and $g_{best}$ is the global best of the objective function.

**Step 6:** Repeat steps 2-5 until the tolerance is less than $10^{-5}$ or the iteration reach to maximum.

The optimization approach is performed in MATLAB on the computer specification of Intel Core i5-10300H, NVIDIA GeForce GTX 1650 and Ram 8 GB DDR4 Bus 2933 MHz.

4. Procedures and system parameters

The case study of the Mass rapid transit Purple Line in Bangkok, Thailand was provided in this paper. With a total distance of 21 km between the first and last passenger stations, Khlong Bang Phai station and Tao Poon station, the line has a total passenger station of 16 and is powered by 750 V DC via its third rail [15, 16]. MATLAB was also used for simulation in this paper. The scenario of the simulation has been separated into two parts: the based case and the case study, MRT Purple Line in Bangkok, Thailand, has been used as a based case for the simulation. Regenerative power from train braking mode for a stop at a passenger station is waste power in the based case simulation without on-board energy storage. Some ways not listed will be used to dispose of such power. As a results as based case, the distribution of energy is calculated in Eq. (12). Besides, the regenerative energy is wasted during train braking expressed as in Eq. (13). In another case of case study, while the train is moving from the passenger station, the on-board equipment can supply power as following in Eq. (14). Consequently, the regenerative power is also capable of being stored in on-board energy storage, then the waste energy can be calculated in Eq. (15). The behaviour of the device in the case study is based on the piece-wise linear control described above.

Tables 1 and 2 shows the traction substation and the train parameters of the MRT Purple Line. Trains have supercapacitors installed on-board. To operate with the train, the connection configuration of the supercapacitor modules would be 15 in series and 10 in parallel. The on-board supercapacitor parameter is shown in Table 3. Fig. 5 depicts the station’s position as well as the gradient of the running route [19].

5. Simulation results and discussions

The simulation result performs with a single-train simulation of MRT Purple Line in MATLAB. The train would make a 25-second stop at the passenger station. The train’s speed profile is determined by the service timetable [19], which may be seen on the website. Each section of the transfer passenger to the next station takes around 2 minutes to complete. The only passenger stations with a maximum transfer

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>2.13</td>
<td>ton</td>
</tr>
<tr>
<td>Max. voltage</td>
<td>765</td>
<td>V</td>
</tr>
<tr>
<td>Min. voltage</td>
<td>705</td>
<td>V</td>
</tr>
<tr>
<td>Max. current</td>
<td>1000</td>
<td>A</td>
</tr>
<tr>
<td>Min. current</td>
<td>0</td>
<td>A</td>
</tr>
<tr>
<td>Energy storage</td>
<td>7.95</td>
<td>kWh</td>
</tr>
<tr>
<td>Resistance</td>
<td>0.9</td>
<td>mΩ</td>
</tr>
<tr>
<td>Eff.</td>
<td>0.90</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 3. On-board supercapacitor parameter

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>153</td>
<td>ton</td>
</tr>
<tr>
<td>Payload mass</td>
<td>75</td>
<td>ton</td>
</tr>
<tr>
<td>Acceleration</td>
<td>1.20</td>
<td>m/s²</td>
</tr>
<tr>
<td>Deceleration</td>
<td>0.90</td>
<td>m/s²</td>
</tr>
<tr>
<td>Max. tractive effort</td>
<td>228.80</td>
<td>kN</td>
</tr>
<tr>
<td>Max. braking effort</td>
<td>168.80</td>
<td>kN</td>
</tr>
<tr>
<td>power auxiliary</td>
<td>270</td>
<td>kW</td>
</tr>
<tr>
<td>Eff. motor/inverter</td>
<td>0.86</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 2. Train parameter

```
Figure. 5 Position (km) vs Height profile (m)

- Gradient profile
- Passenger station
- Traction substation
```
duration of 4 minutes are those from Nonthaburi Civic Center station to the Ministry of Public Health station. The train’s speed profile is set as shown in the Fig 6.

The train started its traveling from the first passenger station at Khlong Bang Phai station to Tao Poon station on the inbound route. After that, the train will start traveling again from Tao Poon station back to Khlong Bang Phai station on the outbound route. Throughout the train journey, power is consumed from the traction substation only in the based case as shown in Fig 7. Therefore, power has a high rate of use. In which in the study case, on-board energy storage is installed to reduce power consumption in travelling.

The piece-wise linear state of charge approach previously described is used to regulate the on-board energy storage performance. The voltage deviation's boundary shown in Table 4. The maximum and minimum deviations are determined using the voltage difference from Eq. (16). The maximum discharge value is obtained by lowering the minimum train voltage difference from the maximum on-board device voltage. The $X$ is the minimum discharge voltage calculated from Eq. (33). The difference between the maximum train voltage and the lowest on-board device voltage is used to obtain the maximum charge value. The minimum deviation of the charge is set to zero. The on-board device's activity is immediate when the voltage difference occurs.

The constraint in Eq. (30) is to find the deviation of the discharge to discharge, based on the conditions in Eq. (29). At the destination, the SOC must be full in 100 %. The PSO optimization in simulation is divided into four case studies based on suitability. They are divided into simulations of the initial SOC of 100 %, 70 %, 50 % and 0 %. From the simulation results, four different discharging deviation values are obtained as shown in Table 5.

The correlation can be seen in Fig. 8 through the SOC curve, which is based on the results collected.
can be shown that when calculated using the optimization approach, it performs better in the range of the SOC percentage of approximately 70%. Because discharge may be done cautiously at 100% initial with the condition that the SOC endpoint must be full. Whereas, with 50% and 0% initial charge, it’s essential to be fully charged at the destination, therefore, there’s only a small amount of discharge along the way. Instead, the range of 70% provides a very flexible range that may be fully charged at the destination during braking.

Fig. 9. depicts the thoughts and actions of the devices in each case. The discharge current is on the positive side. On the other hand, the charge current is the negative current. The results show that the train’s location during the route is at the four, six, eight, ten, and twelve passenger stations, which are passenger stations without a traction substation established. Resulting in a significant voltage variation in the train depending on the usage characteristics, the discharge and charge currents are relatively high.

According to the simulation results, total energy consumption by the train’s is 661.16 kWh. As shown in Table 6. The energy of traction substations can be saved by 2.61 %, 3.49 %, 2.73 % and 2.46 %, respectively, while energy waste can be saved by 8.99 %, 19.37 %, 14.92 % and 16.30 %. At the same time, the energy loss generated in the conductor may be slightly reduced, in which case the initial SOC 70 % is the case with the largest

<table>
<thead>
<tr>
<th>Case</th>
<th>$E_{train}$ [kWh]</th>
<th>$E_{sub}$ [kWh]</th>
<th>Saving [ %]</th>
<th>$E_{loss}$ [kWh]</th>
<th>Saving [ %]</th>
<th>$E_{waste}$ [kWh]</th>
<th>Saving [ %]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Based</td>
<td>661.16</td>
<td>665.64</td>
<td>-</td>
<td>4.48</td>
<td>-</td>
<td>85.34</td>
<td>-</td>
</tr>
<tr>
<td>Initial SOC 100%</td>
<td>661.16</td>
<td>648.29</td>
<td>2.61</td>
<td>4.24</td>
<td>5.57</td>
<td>77.67</td>
<td>8.99</td>
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<tr>
<td>Initial SOC 70%</td>
<td>661.16</td>
<td>642.40</td>
<td>3.49</td>
<td>4.07</td>
<td>9.21</td>
<td>68.81</td>
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<tr>
<td>Initial SOC 50%</td>
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<td>647.49</td>
<td>2.73</td>
<td>4.22</td>
<td>5.93</td>
<td>72.61</td>
<td>14.92</td>
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<tr>
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<td>649.28</td>
<td>2.46</td>
<td>4.28</td>
<td>4.55</td>
<td>71.43</td>
<td>16.30</td>
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</tbody>
</table>

**Figure. 8 State of charge curve, (a) Inbound, (b) Outbound**

**Figure. 9. Current of supercapacitor, (a) Inbound, (b) Outbound**
10 % reduction in energy loss.

6. Conclusion

This paper proposes the use of on-board supercapacitors to mitigate energy distribution and losses in a multi-conductor system of the MRT Purple Line in Bangkok, Thailand. Piece-wise linear SOC control and optimal energy distribution by constraint SOC and deviation minimum discharge voltage of on-board supercapacitor is proposed in case study. As a result, the simulations in all of the presented cases show that in the case of a 70 % initial SOC. The best energy traction substation reduction rate is approximately 3.5 %, reducing energy loss by 10 %. In addition, it can be recharged from braking energy to the device, reducing energy waste by up to 20 %. However, the sizes of the supercapacitors used in the paper are solely chosen based on experience. If the right size is found, the better the results will be. Which will be solved in the further work.

Conflicts of interest

The authors declare no conflict of interest.

Author contributions

Conceptualization, K. Chatwongtong and T. Ratniyomchai; methodology, K. Chatwongtong and T. Ratniyomchai; software, K. Chatwongtong; validation, T. Ratniyomchai; formal analysis, K. Chatwongtong and T. Ratniyomchai; investigation, T. Ratniyomchai; writing—original draft preparation, K. Chatwongtong; writing—review and editing, T. Ratniyomchai; supervision, T. Ratniyomchai.

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